Statistics 512

Major Exam 1

Monday, October 9, 2013

NAME:

Materials permitted:
This exam is open-book and open-note. Anything on paper is allowed. Calculators are allowed. HOWEVER, computers, phones, or any devices capable of wireless communications are not permitted.

Please

• do not open the exam until I say you may

• do not cheat on this exam (do not copy from or communicate with any other person)

Sign Here:

• write your solution clearly and legibly so that I can follow it

• cross out your mistakes; do not erase large quantities of work hand it in

• you have 60 minutes, at which time all papers will be collected.

• unless otherwise stated, use $\alpha = 0.05$. 

A SLR model fit to 25 data points gave \( b_0 = 3, \ b_1 = -2, \ X = 3, \ s_X = 2, \ s_Y = 5, \ \sum e_i^2 = 230. \)

1. [4 points] Find the residual when the response variable is \(-5\) and the explanatory variable is 5. Is the point above or below the line?

Solution: \( \hat{Y}_i = 3 - 2 \times 5 = -7 \quad \Rightarrow \quad e_i = -5 + 7 = 2, \) and the point is above the line.

2. [4 points] Compute root \( MSE \) for this model.

Solution: \( s = \sqrt{MSE} = \sqrt{\frac{230}{23}} = \sqrt{10} \approx 3.16. \)

3. [4 points] Calculate the correlation between \( X \) and \( Y \). Interpret this value.

Solution: \( r = -2 \cdot \frac{2}{5} = -0.8 \). There is a strong negative association (correlation) between \( X \) and \( Y \). If you use \( s_Y = 5.1 \), then \( r \approx -0.79 \)

4. [5 points] What is \( SSR \) for this model?

Solution: \( SSR = SST - SSE \), but \( SST = \frac{SSR}{R^2} = \frac{SSE}{1-R^2} = 639 \quad \Rightarrow \quad SSR = 409. \) If you use \( s_Y = 5.1 \), then \( SSR = 370. \)

5. [4 points] Construct a 95% confidence interval for the slope. [Hint: find \( SS_X \) first].

Solution: \( b_1 \pm t_{23} \times s\{b_1\}. \) But, \( SS_X = (n-1) \cdot s_X^2 = 96 \quad \Rightarrow \quad s\{b_1\} = 0.323. \) The C.I. is \(-2 \pm 2.069 \times 0.323 \quad \Rightarrow \quad (-2.67, -1.33). \)

6. [4 points] Test the hypothesis \( H_0 : \rho \neq 0. \)

Solution: \( H_0 : \rho = 0. \)
\[
t_s = t_{23} = \frac{\sqrt{n-2}}{1-r^2} = -6.2, \quad t_c = 2.069.
\]
Since \( |t_s| > t_c \) we reject \( H_0. \)
or \( t_s = \frac{b_1}{s\{b_1\}} = -\frac{2}{0.323} = -6.2, \) which gives the same conclusion.
or Since 0 falls outside the 95% C.I. for \( b_1 \), we reject \( H_0. \)

7. [5 points] You suspect the slope is less than \(-1.5\), test this claim.

Solution: \( H_0 : \beta_1 = -1.5 \quad vs \quad H_a : \beta_1 < -1.5. \)
\[
t_s = t_{23} = \frac{-2+1.5}{0.323} = -1.55, \quad t_c = -1.714.
\]
Since \( t_s < t_c \) we fail to reject \( H_0. \)

8. [4 points] Find a 95% interval for the mean response when the explanatory is 5.

Solution: \( \hat{Y} \pm t_{23} \times s \times \sqrt{\frac{1}{n} + \frac{(X_h - \bar{X})^2}{SS_X}} \)
\[
-7 \pm 2.069 \times \sqrt{10} \times \sqrt{\frac{1}{25} + \frac{(5-3)^2}{96}} \quad \Rightarrow \quad (-8.87, -5.13). \)

9. [4 points] You are running an experiment and would like to calibrate the explanatory variable, find a calibrated value for the explanatory when the response is \(-5. \)

Solution: \( \hat{X} = \frac{-b_0}{b_1} + \frac{1}{b_1} \cdot Y_h = 1.5 + \frac{5}{2} = 4. \)

10. [2 points] If the power for detecting a change in \( \beta_1 \) is 0.07, what can you say about the \( \alpha \) level?

Solution: \( \alpha \leq 0.07. \)

Good Luck! 😊