Outline

1. CRR Binomial Trees
2. Leisen-Reimer Binomial Tree
3. Flexible Binomial Tree
4. Trinomial Trees
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Exercise

Material: Rouah and Vainberg (2007)

- Recall the Put-Call Parity (at time $t = 0$):

  $$\text{Put price} = \text{Call price} + Ke^{-rT} - S$$

  Compute the price of an European Put Option.

- Modify your code in order to compute the price of ATM American Call option.

- Compute the price of an ATM American Put Option. Can you use the Put-Call Parity here?
CRR Binomial Tree

Recall:

- In the CRR (Cox, Ross and Rubinstein 1979) Binomial tree the increments are defined as:

  \[ u = e^{\sigma \sqrt{T/n}} \]

  \[ d = \frac{1}{u}. \]

- There is a closed formula for the European Call option price:

  \[
  \text{Call Price} = e^{-rT} \sum_{i=0}^{n} \binom{n}{i} q_u^i q_d^{n-i} \max \left( Su^i d^{n-i} - K, 0 \right)
  \]
Construct a VBA function `EuroBinomial` that computes the price of an European Call or Put option with the following parameters:

- Stock price: $S$
- Strike price: $K$
- Risk free rate: $rf$
- Time period: $T$
- Volatility: $\sigma$
- Number of time steps: $n$
- Whether a call or put is priced (String).
Exercise

Construct a VBA function `Binomial` that computes the price of an European or American Call or Put option with the previous parameters and:

- String variable indicating whether an American or European option is priced.
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The CRR Binomial Tree requires many steps to converge to the BS Option formula.

Leisen and Reimer (1996) proposed an improvement using normal approximations. (two different methods)

Only work for odd number of steps.
Take the usual CRR parameters:

\[ u_n = e^{\sigma \sqrt{T/n}} \quad \text{and} \quad d_n = 1/u_n \]

\[ r_n = e^{rT/n} \quad \text{and} \quad p_n = \frac{r_n - d_n}{u_n - d_n} \]

and define the new ones:

\[ u = r_n \frac{p'}{p} \quad \text{and} \quad d = \frac{r_n - pu}{1 - p} \]
where:

\[ p' = h^{-1}(d_1) \]
\[ p = h^{-1}(d_2) \]

\[ h^{-1}(z) = \frac{1}{2} + \frac{\text{Sign}(d_1)}{2} \sqrt{1 - \exp \left[ - \left( \frac{z}{n + \frac{1}{3} + \frac{0.1}{n+1}} \right)^2 \left( n + \frac{1}{6} \right) \right]} \]

\[ d_1 = \frac{\log(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \]
\[ d_1 = \frac{\log(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \]
Exercise

Implement the function $LR$ with the same requirements as $Binomial$. 
Exercise

Compute the price of an European Put Option with parameters:

- \( S = 30 \)
- Strike price: \( K = 30 \)
- Risk free rate: \( rf = 0.05 \)
- Time period: \( T = 0.4167 \)
- Volatility: \( \sigma = 0.3 \)
- Number of time steps: \( n = 10 \) to \( n = 200 \) in steps of size 10.

using the LR and CRR Binomial Tree. Analyze the convergence of both methods.
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Tian (1999) proposed a model that allows CRR tree to be tilted upward or downward. This means that the "center" of the tree increases ($\lambda > 0$) or decreases ($\lambda < 0$) with the time.

The up and down moves are specified by:

$$u = e^{\sigma \sqrt{dt} + \lambda \sigma^2 dt} d$$

The tilt parameter can be chosen to ensure smooth convergence. This value is:

$$\lambda = \log \left( \frac{K}{S} \right) - \frac{(2j_0 - n) \sigma \sqrt{dt}}{n \sigma^2 dt}$$
Where $j_0$ is the integer part of:

$$j_0 = \left\lfloor \frac{\log \left( \frac{K}{S} \right) - n \log \left( d_0 \right)}{\log \left( \frac{u_0}{d_0} \right)} \right\rfloor$$

and $u_0$ and $d_0$ are the up and down moves when $\lambda = 0$ (CRR model).
Exercises

- Implement the function Flexible with the same requirements as Binomial and LR.
- Compute the same European Put option as before. Analyze its convergence and compare it with the Binomial and LR methods.
Extrapolation

This method can be improved by using this extrapolation formula:

\[ \text{Price.ext}(n) = 2\text{Price}(n) - \text{Price}(n/2) \]

Exercise:
Create the function `FlexibleExt` with the extrapolated version of the Flexible Tree. Note that the tree only admits an even number of steps. Analyze the convergence with our European put example.
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Introduced by Boyle (1986). The asset price can go up, down or stay at the same price.

The structure is as follows:
The up movement is given by $u = e^{\sigma \sqrt{2dt}}$ and $d = u^{-1}$.

Probabilities:

$$p_u = \left( \frac{\exp \left( \frac{1}{2} rdt \right) - \exp \left( -\sigma \sqrt{\frac{1}{2} dt} \right)}{\exp \left( \sigma \sqrt{\frac{1}{2} dt} \right) - \exp \left( -\sigma \sqrt{\frac{1}{2} dt} \right)} \right)^2$$

$$p_d = \left( \frac{\exp \left( \sigma \sqrt{\frac{1}{2} dt} \right) - \exp \left( \frac{1}{2} rdt \right)}{\exp \left( \sigma \sqrt{\frac{1}{2} dt} \right) - \exp \left( -\sigma \sqrt{\frac{1}{2} dt} \right)} \right)^2$$

$$p_m = 1 - p_u - p_d.$$
Exercises

- Implement the function Trinomial with the same requirements as Binomial, LR, Flexible and FlexibleExt.
- Compute the same European Put option as before. Analyze its convergence and do a comparison of all the Tree algorithms.