Homework 4 (100pt) Due in class Friday 9/21

1. (8) 1.59 (page 55)

2. (8) 1.67 (page 57)

3. (8) 1.73 (page 58)

4. (16) Case study. Determine the PMF for the following random variables. Identify which one(s) of the random variable is Binomial.

   a) Of all customers purchasing automatic garage-door openers, 75% purchase a chain-driven model. Let $X$ be the number among the next 15 purchasers who select the chain-driven model. Define the PMF of $X$ and decide whether $X$ is Binomially distributed.

   b) A pediatrician wishes to recruit 5 couples, each of whom is expecting their first child, to participate in a new natural childbirth regimen. Let $p = 0.2$ be the probability that a randomly selected couple agrees to participate. Define $X$ be the number of patients the pediatrician asks before she finds 5 couples who agree to participate. (Hint: consider the case where she needs only 1 couple, what is the probability that she asks 1 couple to find 1 couple who agree, how about the probability that she ask 2 couples to find 1 couple? then consider the case where she needs 2 couples, what is the probability that she asks 2 couples to find 2 agree, how about asks 3 couples to find 2 agree… until you find the pattern) Define the PMF of $X$ and decide whether $X$ is Binomially distributed.

   c) Each of 12 refrigerators of a certain type has been returned to a distributor because of an audible, high-pitched, oscillating noise when the refrigerators are running. Suppose that 7 of these refrigerators have a defective compressor and the other 5 have less serious problems. If the refrigerators are examined in random order, let $X$ be the number among the first 6 examined that have a defective compressor. (Hint: for example, the total number of ways to arrange 7 defective refrigerators out of 12 is $\binom{12}{7}$. Suppose there are 2 defective happen in the first 6 examined, these 2 can happen in a total of $\binom{6}{2}$ ways; then the second 6 must contain 7-2=5 defectives, these 5 defectives can happen in a total number of $\binom{6}{5}$ arrangements, what is the probability that 2 defectives are arranged to happen in the first 6 examined?) Define the PMF of $X$ and decide whether $X$ is Binomially distributed.

5. (10) 1.30 (page 41)

6. (10) 1.32 (page 41)
7. \(10\) 1.38 (page 42)

8. (12) Suppose that the reaction time (sec) to a certain stimulus is a continuous random variable with a density function given by

\[ f(x) = \begin{cases} 
\frac{k}{x} & 1 \leq x \leq 10 \\
0 & \text{otherwise}
\end{cases} \]

a) Find the numerical value of \(k\)
b) Find the mean and std of \(x\)
c) Find the mean of \(2x\)
d) Find the mean of \(x^2\).

9. (8) Use normal approximation on Binomial to compute the following questions with continuity correction, suppose \(X \sim \text{Binomial}(500,0.15)\)

a). \(Pr(X < 400)\)
b). \(Pr(200 < X \leq 450)\)

10. (10) Discussion question. Discuss on-line via Blackboard Learn. **Discuss only, no need to turn in your work, grading will be based on your level of participation.**

a). Suppose \(X\) is the cost of an appetizer and \(Y\) be the cost of a main course at a certain restaurant. Suppose a customer always order both courses. The price and probability of an order is given below:

Note \(Pr(X = 5, Y = 10) = 0.2\) means the probability of order a $5 appetizer AND a $10 main course is 20%.

\[Pr(X = 5, Y = 10) = 0.2\]
\[Pr(X = 5, Y = 15) = 0.15\]
\[Pr(X = 5, Y = 20) = 0.05\]
\[Pr(X = 6, Y = 10) = 0.1\]
\[Pr(X = 6, Y = 15) = 0.15\]
\[Pr(X = 6, Y = 20) = 0.1\]
\[Pr(X = 7, Y = 10) = 0.1\]
\[Pr(X = 7, Y = 15) = 0.1\]
\[Pr(X = 7, Y = 20) = 0.05\]

Do you think the order (cost) of appetizer and main course are independent?

b). Suppose \(X \sim \text{Normal}(\mu, \sigma)\), \(Y \sim \text{Normal}(\mu, \sigma)\),

What is \(E(X+X)\), \(\text{Var}(X+X)\), \(E(X+Y)\) and \(\text{Var}(X+Y)\)?