Assignment 8 Answer Keys

Problem 1

(a) This is a random effects experiment because we want to draw inference on the population of looms not just the selected five looms. Our aim is to estimate the variability among all the looms of the textile mill.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>4</td>
<td>0.34160000</td>
<td>0.08540000</td>
<td>5.77</td>
<td>0.0030</td>
</tr>
<tr>
<td>Within</td>
<td>20</td>
<td>0.29600000</td>
<td>0.01480000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>24</td>
<td>0.63760000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

P-value is less than 0.05, reject $H_0$. The looms are not equal in output.

(b) The estimate of variability between looms can be given by $\hat{\sigma}^2 = (0.0854 - 0.0148)/5 = 0.01412$.

(c) The estimate of experimental error variance is: $\hat{\sigma}^2 = 0.0148$.

(d) $F_{0.025,4,20} = 3.51$ and $F_{0.975,4,20} = 1/8.56 = 0.117$. 95% CI for $\sigma^2/(\sigma^2 + \sigma^2)$ is

$$((F_0 - F_{\alpha/2,a-1,N-a})/F_0 + (n - 1)F_{\alpha/2,a-1,N-a}, (F_0 - F_{1-\alpha/2,a-1,N-a})/F_0 + (n - 1)F_{1-\alpha/2,a-1,N-a})$$

$$= ((5.77 - 3.51)/5.77 + 4 \times 3.51, (5.77 - 0.117)/5.77 + 4 \times 0.117)$$

$$= (0.114, 0.906).$$

(e) Need to check assumptions.

Problem 2

SAS uses the unrestricted model. I am analyzing the data assuming a restricted model.

<table>
<thead>
<tr>
<th>Sum of</th>
<th>Source</th>
<th>DF</th>
<th>Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>19</td>
<td>104.8500000</td>
<td>5.5184211</td>
<td>3.68</td>
<td>0.0003</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>40</td>
<td>60.0000000</td>
<td>1.5000000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>59</td>
<td>164.8500000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>operator</td>
<td>1</td>
<td>0.41666667</td>
<td>0.41666667</td>
<td>0.28</td>
<td>0.6011</td>
</tr>
<tr>
<td>part</td>
<td>9</td>
<td>99.01666667</td>
<td>11.00185185</td>
<td>7.33</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>operator*part</td>
<td>9</td>
<td>5.41666667</td>
<td>0.60185185</td>
<td>0.40</td>
<td>0.9270</td>
</tr>
</tbody>
</table>

(a) Here we assume ‘operators’ are fixed effects and ‘parts’ are random effects.

Test for ‘operators’ $H_0: \tau_1 = \tau_2 = 0.$
\( F_0 = \text{MSA/MSAB} = 0.4167/0.6018 = 0.692. \)

P-value is 0.4269. As the P-value is large we fail to reject null hypothesis. The fixed effect ‘operators’ is not significant.

**Test for ‘parts’**  
\( H_0 : \sigma^2_\beta = 0. \)

\( F_0 = \text{MSB/MSE} = 11.00185/1.5 = 7.33. \)

P-value is 0.4269. As the P-value is less than 0.0001 we reject null hypothesis. The random effect ‘parts’ is significant.

**Test for ‘interaction’**  
\( H_0 : \sigma^2_{\alpha\beta} = 0. \)

\( F_0 = \text{MSAB/MSE} = 0.40. \)

As the P-value is very big (much greater than 0.05) we fail to reject \( H_0 \) and conclude that the effect due to ‘interaction’ is not significant.

The variance component estimates are:
\[
\hat{\sigma}^2_\beta = (11.00185 - 1.5)/(2 * 3) = 1.584.
\]
\[
\hat{\sigma}^2_{\tau}\beta = (0.60185 - 1.5)/3 = -0.299(\approx 0).
\]

(b) The exact 95% CI on \( \sigma^2 \):
\[
(df_{MSE}/\chi^2_{0.05/2,40}, df_{MSE}/\chi^2_{1-0.05/2,40}) = (40 \times 1.5/59.34, 40 \times 1.5/24.43) = (1.011, 2.456).
\]