Assignment 6 Answer Keys

Problem 1

(a) The treatment sum of squares can be calculated by

\[
SS_{\text{Treatment}} = b \sum_{i=1}^{a} \bar{y}_i^2 - N \bar{y}^2,
\]

\[
= 5 \cdot (5.40^2 + 5.80^2 + 10^2 + 9.80^2) - 20 \cdot 7.75^2
\]

\[
= 92.95
\]

Then the \(F\) statistic for testing the treatment effect is

\[
F = \frac{SS_{\text{Treatment}}}{(a - 1)}MS_E = \frac{92.95/3}{6.275} = 4.938,
\]

which is greater than \(F_{0.05,3,12} = 3.49\), the 95\% percentile of distribution \(F_{3,12}\). Hence I conclude that there are differences among the four cooling temperatures.

(b) The critical distance for Tukey’s pairwise comparisons method is

\[
CD = q_{\alpha,a,(a-1)(b-1)} \sqrt{MS_E/b} = q_{0.05,4,12} \sqrt{6.275/5} = 4.705.
\]

The maximum difference among the four temperature groups is 10 - 5.40 = 4.60, which is less than the above critical difference. Hence Tukey’s method detects no significant differences among the four temperature groups. As shown below, my computation is consistent with the SAS output.

Tukey’s Studentized Range (HSD) Test for \(y\)

NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

<table>
<thead>
<tr>
<th>Alpha</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Degrees of Freedom</td>
<td>12</td>
</tr>
<tr>
<td>Error Mean Square</td>
<td>6.275</td>
</tr>
<tr>
<td>Critical Value of Studentized Range</td>
<td>4.19852</td>
</tr>
<tr>
<td>Minimum Significant Difference</td>
<td>4.7035</td>
</tr>
</tbody>
</table>

Means with the same letter are not significantly different.

Tukey Grouping | Mean | N | tmp |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10.000</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>A</td>
<td>9.800</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>A</td>
<td>5.800</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>5.400</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

(c) The contrasts are

\[
C_1 = (1, -1, 0, 0),
\]

\[
C_2 = (0, 0, 1, -1),
\]

\[
C_3 = (1, 1, -1, -1),
\]
where $C_1$ is used to test the difference between temperatures $5^\circ$ and $10^\circ$, $C_2$ to test the difference between temperatures $15^\circ$ and $20^\circ$, and $C_3$ to test the difference between lower temperatures ($5^\circ$, $10^\circ$) and higher temperatures ($15^\circ$, $20^\circ$). It’s obviously that they form a complete set of orthogonal contrasts.

(d) The SAS output for testing the above contrasts is shown below.

<table>
<thead>
<tr>
<th>Contrast</th>
<th>DF</th>
<th>Contrast SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>0.40000000</td>
<td>0.40000000</td>
<td>0.06</td>
<td>0.8049</td>
</tr>
<tr>
<td>C2</td>
<td>1</td>
<td>0.10000000</td>
<td>0.10000000</td>
<td>0.02</td>
<td>0.9016</td>
</tr>
<tr>
<td>C3</td>
<td>1</td>
<td>92.45000000</td>
<td>92.45000000</td>
<td>14.73</td>
<td>0.0024</td>
</tr>
</tbody>
</table>

Contrasts $C_1$ and $C_2$ are not significant, while contrast $C_3$ is significant. This confirms the belief of the company that there is a jump in the strength at some temperature ($12.5^\circ$) between $10^\circ$ and $15^\circ$.

Problem 2

(a) To do Tukey’s Additivity test, I first fit an additive model and obtain the predicted values $\hat{y}_{ij}$, then add $q_{ij} = \hat{y}_{ij}^2$ to the predictor set and fit another model. Then Tukey’s test is equivalent to the significance test of $q$. Here is the SAS output

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>1</td>
<td>8.19424514</td>
<td>8.19424514</td>
<td>3.85</td>
<td>0.1070</td>
</tr>
</tbody>
</table>

Since $q$ is not significant ($p$-value $= 0.1070$), I conclude that there’re no interaction effects between detergent and stain, and the additive model is valid.

Problem 3

(a) The hypotheses are

$H_0$: There is no difference between the four assembly methods, or symbolically, $\tau_A = \tau_B = \tau_C = \tau_D$.

$H_1$: There is a difference between the four assembly methods, or symbolically, $\tau_A, \tau_B, \tau_C$ and $\tau_D$ are not all equal.

Let’s look at the ANOVA table output from SAS (I replaced the line for the model SS by lines for the two block SS and the treatment SS).

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>ord</td>
<td>3</td>
<td>18.50000000</td>
<td>6.16666667</td>
<td>3.52</td>
<td>0.0885</td>
</tr>
<tr>
<td>opt</td>
<td>3</td>
<td>51.50000000</td>
<td>17.16666667</td>
<td>9.81</td>
<td>0.0099</td>
</tr>
<tr>
<td>trt</td>
<td>3</td>
<td>72.50000000</td>
<td>24.16666667</td>
<td>13.81</td>
<td>0.0042</td>
</tr>
<tr>
<td>Error</td>
<td>6</td>
<td>10.50000000</td>
<td>1.75000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>15</td>
<td>153.0000000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Since p-value for treatment effect is small (= 0.0042), I conclude that there is a difference between the four assembly methods.

(b) The treatment effects $\tau_j, j = A, B, C, D$ are estimated by
$$\hat{\tau}_j = \bar{y}_j - \bar{y}_\cdot, \quad j = A, B, C, D.$$ The overall mean $\bar{y}_\cdot$ and the treatment group means $\bar{y}_j$ can be obtained from the SAS output of PROC GLM.

<table>
<thead>
<tr>
<th>R-Square</th>
<th>Coeff Var</th>
<th>Root MSE</th>
<th>y Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.931373</td>
<td>12.90610</td>
<td>1.322876</td>
<td>10.25000</td>
</tr>
</tbody>
</table>

Plugging in these values, I get the four treatment effect estimates as below

<table>
<thead>
<tr>
<th>$\hat{\tau}_A$</th>
<th>$\hat{\tau}_B$</th>
<th>$\hat{\tau}_C$</th>
<th>$\hat{\tau}_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2.75</td>
<td>−1.00</td>
<td>3.00</td>
<td>0.75</td>
</tr>
</tbody>
</table>

(c) The critical distance for Tukey’s pairwise comparisons method is
$$CD = q_{0.05,4,6} \sqrt{MSE/p} = q_{0.05,4,6} \sqrt{1.75/4} = 3.24.$$ The four treatment means are ordered as $13.25 > 11.00 > 9.25 > 7.50 (C > D > B > A)$. After computing differences following this order and comparing them with the critical distance, I reach the following conclusion.

- The pairs of assembly methods which have significantly different effects are $(C, B), (C, A), (D, A)$.
- The pairs of assembly methods whose effects are not significantly different are $(C, D), (D, B), (B, A)$.

(d) The diagnostic plots in Figure 1 are: normal probability Q-Q plot, plot of residuals versus assembly methods (treatment), plot of residuals versus assembly orders (row block), plot of residuals versus operators (column block), and plot of residuals versus predicted values.

The normal Q-Q plot shows that the normality assumption is valid. And there are no potential outliers or influential points in the plots. Only the plot of residuals against predicted values shows some curvilinearity, but this is not enough to question on the additivity assumption since our sample size is small.

Problem 4

(a) This is a $4 \times 4$ Graeco-Latin square design. It superimposes on the Latin square of 4 assembly methods another Latin square of 4 workplaces. And these two Latin squares are orthogonal to each other, that is, each assembly method in the first Latin square is paired with each workplace in the second Latin square exactly once.

(b) The ANOVA table from SAS is as follows (again, I replaced the line for the model SS by lines for the block SS and the treatment SS).
The $p$-value for the treatment effect is large ($= 0.1669$), so I conclude that the four assembly methods are not different.

(c) My conclusion here is inconsistent with that from Problem 1. First, our data are different from those in Problem 1 and seem to have less variation due to assembly methods (treatment SS here, 7.5, is only about 1/10 of that in Problem 1, 72.5). Second, the Graeco-Latin square design reduces the

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Source} & \text{DF} & \text{Sum of Squares} & \text{Mean Square} & \text{F Value} & \text{Pr > F} \\
\hline
\text{ord} & 3 & 0.50000000 & 0.16666667 & 0.02 & 0.9960 \\
\text{opt} & 3 & 19.00000000 & 6.33333333 & 0.69 & 0.6157 \\
\text{trt} & 3 & 95.50000000 & 31.83333333 & 3.47 & 0.1669 \\
\text{wp} & 3 & 7.50000000 & 2.50000000 & 0.27 & 0.8429 \\
\hline
\text{Error} & 3 & 27.5000000 & 9.1666667 & & \\
\hline
\text{Corrected Total} & 15 & 150.0000000 & & & \\
\hline
\end{array}
\]
degree of freedom for $MS_E$ from 6 to 3, which may cause the $F$ test for the treatment effect less sensitive.