STAT 350 Exam 2 (Version A)  
Spring 2015

SOLUTION

Instructor (circle one): Ayu Findsen Sellke Troisi

Class Time:  
11:30 AM  12:30 PM  1:30 PM  
2:30 PM  3:30 PM  4:30 PM  ONLINE

Instructions:
1. You are expected to uphold the honor code of Purdue University. It is your responsibility to keep your work covered at all times. Anyone cheating on the exam will automatically fail the course, and will be reported to the Office of Dean of Students.
2. Please alert proctors if you observe any cheating during the exam. We highly appreciate it.
3. It is strictly prohibited to smuggle this exam outside. Your exam will be returned to you after it is graded.
4. You may have one double-sided 8.5 in x 11 in crib sheet to take this test. The crib sheet can be handwritten or typed.
5. The only materials that you are allowed are your calculator, writing utensils and erasers and your crib sheet. If you bring any other papers in to the exam, you will get a zero on the exam. We will provide scratch paper if you need more room.
6. Leave all your belongings except those permitted for the exam in the front of the room. We are not responsible for any loss.
7. If you share your calculator or use a cell phone, you will get a zero on the exam.
8. Breaks (including bathroom breaks) during the exam are not allowed. If you leave the exam room, you must turn in your exam and you will not be allowed to come back.
9. You must show ALL your work to obtain full credit. An answer without showing any work may result in zero credit.
10. All numeric answers should have two decimal places except answers from the z-table should have four decimal places.
11. If your work is not readable, it will be marked wrong.
12. After you complete the exam, please turn in your exam as well as your crib sheet, tables and any scrap paper. Please be prepared to show your Purdue picture ID.

Your exam is not valid without your signature below.

STUDENT: I attest here that I have followed the instructions above honestly while taking this test and that work submitted is my own, produced without assistance from books, other people, notes other than my own crib sheets, or other aids.

Signature of Student: __________________________________________
<table>
<thead>
<tr>
<th>Points Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1 (30 points)</td>
</tr>
<tr>
<td>Problem 2 (25 points)</td>
</tr>
<tr>
<td>Problem 3 (20 points)</td>
</tr>
<tr>
<td>Problem 4 (30 points)</td>
</tr>
<tr>
<td>Total (105 / 100 )</td>
</tr>
</tbody>
</table>
0. **(6 points)** What fun outdoor activities do you like to do? (please list two)

1. **(24 points)** Multiple Choice Questions (Circle Only One Answer):

   1.1 **(4 pts)** Suppose that a random sample of 50 bottles of a particular brand of cough syrup is selected and the alcohol content of each bottle is determined. Suppose that the resulting 95% confidence interval for the mean alcohol content is (7.8, 9.4). We may conclude that
   a. There is a 95% chance that \( \mu \) is between 7.8 and 9.4.
   b. If the process of selecting a sample of size 50 and then computing the corresponding 95% interval is repeated a very large number of times, approximately 95% of the resulting intervals will include \( \mu \).
   c. We can be highly confident that 95% of all bottles of this type of cough syrup have an alcohol content that is between 7.8 and 9.4.
   d. All of the above

   1.2 **(4 pts)** An agricultural researcher plants 25 plots with a new variety of yellow corn. Assume that the yield per acre for the new variety of yellow corn follows a Normal distribution with an unknown mean \( \mu \) of and a standard deviation of \( \sigma = 10 \) bushels per acre. Which of the following would produce a confidence interval with a smaller margin of error than the 90% confidence interval?
   a. Compute a 99% confidence interval rather than a 90% confidence interval, because a higher confidence level will result in a smaller margin of error.
   b. Plant only five plots rather than 25, because five are easier to manage and control.
   c. Plant 10 plots rather than 25, because a smaller sample size will result in a smaller margin of error.
   d. Plant 100 plots rather than 25, because a larger sample size will result in a smaller margin of error.

   1.3 **(4 pts)** The manager of an automobile dealership is considering a new bonus plan to increase sales. Currently, the mean sales rate per salesperson is 5 automobiles per month. The correct set of hypotheses to test the effect of the bonus plan is
   a. \( H_0: \mu = 5 \) vs \( H_a: \mu > 5 \)
   b. \( H_0: \mu > 5 \) vs \( H_a: \mu = 5 \)
   c. \( H_0: \bar{x} = 5 \) vs \( H_a: \bar{x} > 5 \)
1.4  **(4 pts)** Suppose we are testing $H_0: \mu = 50$ and $H_a: \mu \neq 50$ for a normal population with $\sigma = 6$. The 95% confidence interval for the mean is $(51.3, 54.7)$. Then
   a. the P-value for the test is greater than 0.05
   b. the P-value for the test is less than 0.05
   c. the P-value for the test could be greater or less than 0.05. It can't be determined without knowing the sample size.

1.5  **(4 pts)** A radio station wants to know if residents in their area are in favor of a proposed tax increase. They invite listeners to call in to respond to the poll. Of the 800 who responded, 645 were against the tax. The station calculated a 95% confidence interval and declared "Between 77.9% and 83.4% of residents oppose the new tax."
   a. This is a valid interval, so the station must be right.
   b. We can't say for certain that between 77.9% and 83.4% of residents oppose the new tax because it is a 95% confidence interval.
   c. Because of the way the poll was conducted, the results are invalid.

1.6  **(4 pts)** A study was conducted to compare five different training programs for improving endurance. 40 subjects were randomly divided into five groups of eight subjects in each group. A different training program was assigned to each group. After 2 months, the improvement in endurance was recorded for each subject. A one-way ANOVA is used to compare the five training programs, and the resulting p-value is 0.02. At a significance level of 0.05, what is the appropriate conclusion about mean improvement in endurance?
   a. The average amount of improvement appears to be the same for all five training programs.
   b. The average amount of improvement appears to be different for each of the five training programs.
   c. It appears that at least one of the five training programs has a different average amount of improvement.
   d. One training program is significantly better than the other four

2.  **(25 points)** A pharmaceutical company has developed a new drug to reduce cholesterol. The company gives the new drug to a simple random sample of 60 people from the population
of people with high cholesterol. The reduction in cholesterol level after one month of use was recorded for each individual in the sample, resulting in a sample mean reduction of 24 mg/dl. The population standard deviation of cholesterol reduction is known to be 15 mg/dl.

(a) (10 pts) Find a (two-sided) 99% confidence interval for the true mean cholesterol reduction if the drug were used by people with high cholesterol. Be sure to interpret your interval in context.

(3 pts) $z^* = 2.576$ (also accept 2.57, 2.58) for $C=99\%$

(3 pts) $24 \pm 2.576 \times \frac{15}{\sqrt{60}}$

(1 pt) $= 24 \pm 4.99$

M.E. = 4.98 if $z^* = 2.57$

M.E. = 5.00 if $z^* = 2.58$

(1 pt)

(1 pt) $(19.01, 28.99)$ for $z^* = 2.576$

$(19.02, 28.98)$ for $z^* = 2.57$

$(19.00, 29.00)$ for $z^* = 2.58$

(2 pts) We are 99% confident that the true mean cholesterol reduction if the drug were used by people with high cholesterol is between 19.01 mg/dl and 28.99 mg/dl.

(b) (5 pts) How many samples should we take if we want the margin of error to be 3 mg/dl with 99% confidence?

(2 pts) $ME = 2.576 \times \frac{15}{\sqrt{n}} = 3$

$sqrt(n) = 2.576 \times 15 / 3 = 12.88$

(1 pt) $n = 165.89$

$n = 165.12$ for $z^* = 2.57$

$n = 166.41$ for $z^* = 2.58$

(2 pts) round up $=> n = 166$ (for $z^* = 2.57$ and 2.576)

$N = 167$ for $z^* = 2.58$

2. (cont’d)
(c) (10 pts) A regulatory agency will recommend the new drug for use if there is convincing evidence that the mean reduction in cholesterol level after one month of use is more than 20 mg/dl, because a mean reduction of this magnitude would be greater than the mean reduction
for the current most widely used drug. Use the four step procedure to test the following hypotheses with $\alpha = 0.05$:

$$H_0: \mu = 20 \quad \text{versus} \quad H_a: \mu > 20$$

where $\mu$ represents the population mean reduction in cholesterol level for the new drug.

(1 pts) Step 1: $H_0: \mu = 20 \quad \text{versus} \quad H_a: \mu > 20$

(3 pts.) Step 2:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{24 - 20}{15/\sqrt{60}} = 2.07$$

(3 pts.) Step 3:

$$\text{P-value} = \text{P}(Z > 2.07) = 0.0192$$

(3 pts.) Step 4

Reject $H_0$ because p-value = 0.0192 < 0.05

The data strongly supports ($P = 0.0192$) the claim that the true mean reduction $\mu$ in cholesterol level after one month of use is more than 20 mg/dl
Hans and Franz ‘Test’ their Heart ‘Power’!: Hans and Franz decide to have a 1 week (7 day) ridiculously heavy jump rope competition against one another. The rules are that each will start 100 rope jumps at the same time of 5pm each day together and record the duration/time in seconds until completion. On the seventh day, Franz had an injury, so had to pull out and not take part. However, since the brothers are still so supportive of one another’s weight training goals, Franz did shout encouraging slogans while Hans jumped rope on the seventh day. The data measured to the nearest tenth of a second of precision are below:

<table>
<thead>
<tr>
<th>Day</th>
<th>Hanz</th>
<th>Franz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>102.9</td>
<td>98.9</td>
</tr>
<tr>
<td>2</td>
<td>84.6</td>
<td>82.7</td>
</tr>
<tr>
<td>3</td>
<td>103.5</td>
<td>99.5</td>
</tr>
<tr>
<td>4</td>
<td>87.4</td>
<td>84.7</td>
</tr>
<tr>
<td>5</td>
<td>94.7</td>
<td>95.2</td>
</tr>
<tr>
<td>6</td>
<td>95.1</td>
<td>93.0</td>
</tr>
<tr>
<td>7</td>
<td>89.0</td>
<td>NUL</td>
</tr>
<tr>
<td>Mean</td>
<td>93.88571</td>
<td>92.333...</td>
</tr>
</tbody>
</table>

| St. Dev. | 7.747774| 7.130404|

a. (10 points) Franz won the competition! However, that does not necessarily mean that Franz is a better heavy jump roper than Hans. Construct a 95% confidence interval for the difference of their true mean time to complete the 100 jumps based on ALL the data above.

(1 pt) Use Two-sample T procedure.
1 -- Hans
2 -- Franz

(2 pts) A 95% confidence interval for $\mu_1 - \mu_2$ is given by:

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

(2 pts) The critical value for 95% confidence is $t^* = 2.571$ based on DF = 5

(2 pts) The 95% confidence interval for $\mu$ is therefore

$$1.55 \pm 2.571 \sqrt{\frac{2.25^2}{7} + \frac{2.13^2}{6}} = 1.55 \pm 2.571 \times 4.13$$

$$= 1.55 \pm 10.62$$

(1 pt) which is the interval (-9.07, 12.17) minutes.

(2 pts) We can be 95% confident that the difference of their true mean time to complete the 100 jumps is between -9.07 minutes and 10.62 minutes.

3. (cont’d)
b. (5 points) Utilize the plot above of the data to argue whether or not the test was appropriate or not.

According to the QQ plots, each sample seems to be approximately Normal as the points are close to straight lines. Therefore, the two-sample T procedure was appropriate.

c. (5 points) If Franz had not encountered an injury or Hans opted to not jump on the seventh day without his brother, what would have been the appropriate test as to whether Franz’s true mean time to complete the 100 jumps is faster than Hans? Explain.

Match Pairs T procedure would be more appropriate. Explanation varies.
4. (30 points) An experimenter was interested in investigating the effects of two stimulant drugs (labeled A and B). She divided 25 rats equally into 5 groups (placebo, Drug A low, Drug A high, Drug B low, and Drug B high) and, 20 minutes after injection of the drug, recorded each rat’s activity level (higher score is more active). The following table summarizes the results:

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Sample Mean</th>
<th>Sample Standard Deviation (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Placebo</td>
<td>11.80</td>
<td>4.2</td>
</tr>
<tr>
<td>Low A</td>
<td>15.25</td>
<td>3.6</td>
</tr>
<tr>
<td>High A</td>
<td>18.55</td>
<td>3.2</td>
</tr>
<tr>
<td>Low B</td>
<td>16.15</td>
<td>2.8</td>
</tr>
<tr>
<td>High B</td>
<td>17.10</td>
<td>3.5</td>
</tr>
</tbody>
</table>

(a) (5 pts) Plot the means versus the type of treatment.

(b) (4 pts) Is it reasonable to assume that the variances are equal? Explain.

(1 pts) Yes,
(3 pts) $4.2/2.8 < 2$

(c) (6 pts) Find the pooled standard deviation. What is the value of MSE?

(2 pts) $\text{MSE} = \text{pooled variance}$
(2 pts) $= 4*(4.2^2+3.6^2+3.2^2+2.8^2+3.5^2)/20$
(2 pts) $= 12.186$

(2 pts) $\text{Pooled SD} = \sqrt{12.186} = 3.49$
(d) (8 pts) SSG is 128.37. Find the value of the F statistic. Give the numerator and denominator degrees of freedom

(2 pts) DF1 = I - 1 = 4,
(2 pts) DF2 = N - I = 20

(2 pts) MSG = SSG/DFG = 128.37/4 = 32.093
MSE = 12.186
(2 pts) F = MSG/MSE = 32.093/12.188 = 2.63

(e) (4 pts) The p-value is 0.065. At a significance level of 0.05, what is the appropriate conclusion?

We fail to reject H0. Drug A, drug B, and placebo have the same effect in rats’ activity level.

(f) (3 pts) Should a multiple comparison test be performed in this situation? Explain your answer in one sentence.

No, we should not perform a multiple comparison test. We only perform a multiple comparison test if H0 is rejected.