STAT 512 – Homework #4
Due Date: Friday, February 11th, Beginning of Class

Reminder: Please include your name, instructor, homework number, due date, and course on the first page of your completed homework.

Guidelines: Please remember that neatness is important! Use complete sentences and/or show work (mathtype / equation editors) as necessary. Answers that are not supported by reasoning will not be graded. Homework must always be stapled if it is longer than one page. If it is not stapled, only the first page will be graded. References to KNNL, refer to the required text (Kutner, Nachtsheim, Neter, Li; 5th edition). You will not need to use SAS for this homework.

1. (10 pts) Please locate (in the literature pertaining to your field if possible) an application of regression. Provide a well written, concise review of the article (1-2 paragraphs at most). State the main goals of the article and the variables that were being investigated. Notice how the results are presented and look to see if there is any mention of checking the model assumptions. Discuss the main findings. Please include a copy of the article as an attachment at the back of your homework.

2. (2pts) Show that $R^2 \left(1 - R^2\right) = \frac{SSR}{SSE}$.

3. (3 pts) In running the analysis for a simple linear regression, you have $n = 22$ observations and find that $R^2 = 0.64$.

   (a) Use your results from Problem 2 to calculate the F-statistic for testing $H_0 : \beta_1 = 0$.

   (b) What is the conclusion of this significance test for $\alpha = 0.05$? Note that $F_{.95}(1, 20) = 4.35$.

4. (8 pts) Based on the following data set: X: 2 4 6 8 10 Y: 1 2 5 7 9

   (a) Construct the design matrix ($X$) and its transpose ($X'$)

   (b) Given that $(X'X)^{-1} = \begin{bmatrix} 1.1 & -0.15 \\ -0.15 & 0.025 \end{bmatrix}$, find $b = (X'X)^{-1}X'Y$. (If you have trouble with matrix multiplication, see Lecture 8 or KNNL 5.3). In this vector, identify $b_0$ and $b_1$.

   (c) Given $(X'X)^{-1}$ above and MSE=0.23, then find $s^2 \{b\}$. In this matrix, identify $s^2 \{b_0\}$ and $s^2 \{b_1\}$.

   (d) Construct the 95% confidence interval for $\beta_1$ and conduct the test for $H_0 : \beta_1 = 0$. Note that $t_{.975}(3) = 3.182$. ***Corrected from previous version.***
5. (2 pts) KNNL 6.1 (a)

6. (10 pts) A researcher in a scientific foundation wished to evaluate the relation between annual salaries (Y, in thousand dollars) of mathematicians and an index of work quality (X₁), number of years of experience (X₂), and an index of publication success (X₃). Data were collected for a sample of 24 mathematicians. A multiple linear regression was employed with quality, experience, and publications as the explanatory variables and salary as the response variable.

(a) Complete the following ANOVA table for the regression analysis.

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<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
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<td>__</td>
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<td>__</td>
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<tr>
<td>Error</td>
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</tr>
</tbody>
</table>

(b) Using your results from (a), calculate $R^2$ and adjusted $R^2_a$. Suppose we also have data on an index of teaching quality (X₄). How would adding this new predictor variable to the model affect the $R^2$ and $R^2_a$?

(c) State the null and alternative hypotheses for the F-test as well as your conclusion in sentence form. Does this test give us any information about which of the predictors (quality, experience, and publications) are important?