Problems 1

**Data:**
- \( S = 45 \), \( C = 24.45 \), \( P = 1.90 \)

**Part A:**
- \( n^A_1 = 2 \), \( n^A_2 = 1 \)

**Part B:**
- \( n^B_1 = 2 \), \( n^B_2 = -3 \)

**Unknown:**
- \( \Delta_{Pal} = \frac{\Delta_{Pal}}{P} \)

**We have:**
- \( \Delta^A = 5.0 = \frac{2 \times 4.45}{2 \times 4.45 + 1.9} \Delta_{Cell} + \frac{1.9}{2 \times 4.45 + 1.9} \Delta_{Pal} \quad - (1) \)

- \( \Delta^B = 3.4 = \Delta_{Cell} - 3 \Delta_{Pal} \quad - (2) \)

(3) \( \Delta_{Pal} = \frac{S \Delta_{Pal}}{P} \), \( \Delta_{Cell} = \frac{S \Delta_{Cell}}{C} \) - (4)

**Solution:**
- Plug (3) into (4) into (2):

\[
(2') \quad \frac{0.197}{45} \Delta_{Cell} = \frac{0.126}{45} \Delta_{Pal} = 3.41.
\]

(2') with (1) form a linear system on \( \Delta_{Cell} \) and \( \Delta_{Pal} \).

Solve this system:

\[
0.082 \Delta_{Cell} + 0.18 \Delta_{Pal} = 5.0 \quad \text{and} \quad 0.197 \Delta_{Cell} - 0.126 \Delta_{Pal} = 3.41 \quad \Rightarrow \quad \Delta_{Cell} = \frac{3.41 + 0.126 \Delta_{Pal}}{0.197}
\]

\[
0.204 \Delta_{Pal} = -9.152
\]

\( \Rightarrow \quad \Delta_{Pal} = -43.03 \)
Problem 2.

\[-Q_{pu} = \frac{D_{pu} \Sigma}{P} \Rightarrow -6 = \frac{-0.46 \times S_0}{7.5}\]

\[\Rightarrow S_0 = 97.82604\]

Also,

\[Q_{cell} = \frac{\Delta_{cell} S_0}{C} \Rightarrow 18 = \frac{\Delta_{cell} \times 97.82604}{C}\]

Recall that

\[\Delta_{cell} = e^{-\sigma T} N(d_i) = N(d_i)\]

\[\Delta_{pu} = e^{\sigma T} N(-d_i) = -N(-d_i) \Rightarrow \Delta_{cell} = 1 - N(-d_i) = 1 + \Delta_{pu}\]

So,

\[\Delta_{cell} = 0.59\]

Finally,

\[C = \frac{0.59 \times 97.82604}{18} = 2.9397\]
Problem 3.

Several months ago, she sold the call option on the 100 shares and bought the replicating portfolio. Thus, the transactions at time 0 and the associated cash flows are as follows:

<table>
<thead>
<tr>
<th>Transactions</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell call</td>
<td>100 x 8.88 = 888</td>
</tr>
<tr>
<td>Buy 100 ( \Delta ) shares of stock</td>
<td>-100 x 0.799 x 40 = -3176</td>
</tr>
<tr>
<td>Borrow remaining money</td>
<td>+2288</td>
</tr>
<tr>
<td>Net cashflow</td>
<td>0</td>
</tr>
</tbody>
</table>

Now, she closes her positions which result in the following cashflow:

<table>
<thead>
<tr>
<th>Transactions</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy back call</td>
<td>-100 x 14.42 = -1442</td>
</tr>
<tr>
<td>Sell 100 shares</td>
<td>100 x 0.799 x 50 = 3970</td>
</tr>
<tr>
<td>Pay back loan</td>
<td>-2288 x e(^{-rt}) \quad t = \text{elapsed time from the time the portfolio was started and now}</td>
</tr>
<tr>
<td>Net cashflow</td>
<td>-2288 x e(^{-rt}) + 3970 - 1442</td>
</tr>
</tbody>
</table>

We need to find e\(^{-rt}\).

Note that, by call/put parity,

\[ c_{\text{now}} - p_{\text{now}} = s_{\text{now}} - K e^{-r(t-t)} \Rightarrow 50 - K e^{-r} e^{-rt} = 14.16 \]

\[ c_{\text{bet}} - p_{\text{bet}} = s_{\text{bet}} - K e^{-r} \Rightarrow 40 - K e^{-r} = 7.25 \]

Then, \[ 50 - (40 - 7.25) e^{-r} = 14.16 \Rightarrow e^{-r} = 1.0943 \]

Finally, her profit/loss is -2288 x 1.0943 + 3970 - 1442 = 241.
Problem 4.

8 months after purchasing the option, the remaining time to expiration = 4 months.

\[ d_1 = \frac{\ln(85/75) + (0.05 - 0 + \frac{1}{2} \times 0.26^2) \times 4/12}{0.26 \sqrt{4/12}} = 1.019888 \approx 1.02, \quad N(d_1) \approx 0.8461, \]

\[ d_2 = d_1 - \sigma \sqrt{T} = 1.019888 - 0.26 \sqrt{4/12} = 0.869777 \approx 0.87, \quad N(d_2) \approx 0.8078 \]

At time of purchase,

\[ C = SN(d_1) - Ke^{-rT}N(d_2) \approx 85 \times 0.8461 - 75e^{-0.05 \times (4/12)} \times 0.8078 = 12.3349 \]

Hence, 8-month holding profit is \(12.3349 - 8e^{0.05 \times 8/12} = 4.0637 \approx 4.06.\)
Problem 5.

(a) We first compute the delta of the initial portfolio:

\[ \Delta_{\text{initial}} = \Delta_{\text{butterfly}} = (1)(-0.8) + (3)(-0.1) - (2)(-0.4) \]
\[ = -0.1. \]

Next, to delta-hedge, we buy \( n \) shares of the stock to make the final portfolio's delta equal to 0:

\[ \Delta_{\text{final}} = \Delta_{\text{initial}} + n = 0 \implies n = -\Delta_{\text{initial}} = -(0.1) \]
\[ = 0.1. \]

We need to buy 0.1 shares.

(b) At time \( t=0 \), we perform the following transactions:

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy 1 40-strike</td>
<td>- 2.0</td>
</tr>
<tr>
<td>Buy 3 60-strike</td>
<td>- 12.0</td>
</tr>
<tr>
<td>Sell 2 50-strike</td>
<td>2 \times 5 = 10.0</td>
</tr>
<tr>
<td>Buy 0.1 shares</td>
<td>-0.1 \times 48 = -4.8</td>
</tr>
<tr>
<td>Borrow from bank</td>
<td>8.8</td>
</tr>
<tr>
<td>Net cashflow</td>
<td>0</td>
</tr>
</tbody>
</table>

After 1 month, the marking-to-market P/L is

\[
P/L = \underbrace{1.8}_{\text{sell the 40-strike put}} + \underbrace{10.0}_{\text{sell the 60-strike put}} - \underbrace{2 \times 4.0}_{\text{buy base}} + \underbrace{0.1 \times 50 - 8.8}_{\text{sell the 2 50-strike put, 0.1 shares, pay back loan}} \]
\[= -0.0367. \]
Problem 6.

This is really a put-call parity question, but in the context of delta hedging. Let $t_1$ be the time the option was bought, $t_2$ the time it was sold, and $T$ margin expiry. In the following lines, we calculate the factor to accumulate an investment from time $t_1$ to time $t_2$.

\[
C(S, K, t) - P(S, K, t) = S - Ke^{-rt}
\]

4.25 - 8.50 = -4.25 = 40 - $Ke^{-r(T-t)}$

9.30 - 5.30 = 4.0 = 50 - $Ke^{-r(T-t)}$

$Ke^{-r(T-t)} = 44.25$

$Ke^{-r(T-t)} = 46.0$

\[
e^{r(t_2-t_1)} = \frac{44.25}{46.0} = 1.03815
\]

At time $t_1$, the investor purchased a call and sold short 0.3 shares of stock. The investment at time $t_1$ was $4.25 - 0.3(40) = -7.75$. This investment is worth $9.30 - 0.3(50) = -5.70$ at time $t_2$. The profit is the proceeds of $-5.70$ minus the original investment $-7.75$ accumulated at interest.

\[-5.70 - (-7.75)(1.03815) = 0.358\]

Finally, multiply all the above answers by 50.

Problem 7.

\[C^2(S) + \Delta \left( \text{Change in Stock Price} \right) \]

\[18.3571 + (1.7216)(1) = 19.0877\]

\[C^2(S) + \Delta \left( \text{Change in Stock Price} \right) + \frac{\Delta}{2} \left( \text{Change in Stock Price} \right)^2 \]

\[= 18.3571 + (1.7216)(1) + (0.112)(1.7216)(1)^2\]

\[= 19.1443\]

\[C^2(S) + \Delta \left( \text{Change in Price} \right) + \frac{\Delta}{2} \left( \text{Change in Price} \right)^2 + \Theta \left( \text{Number of Days} \right) \]

\[= 18.3571 + (1.7216)(1) + 1.012(0.16)(1) + (-0.0256)(1)\]

\[= 19.0887\]
Problem 8.

(a) 
\[
\text{Cov}(\bar{Z}(2), \bar{Z}(3)) = \text{Cov}(\bar{Z}(2), \bar{Z}(3) - \bar{Z}(2) + \bar{Z}(2)) \\
= \text{Cov}(\bar{Z}(2), \bar{Z}(3) - \bar{Z}(2)) + \text{Cov}(\bar{Z}(2), \bar{Z}(2)) \\
= 0 + \text{Var}(\bar{Z}(2)) \\
= 2
\]

(b) 
All we saw is that \(\int_0^t \bar{Z}(t) \, dt = \frac{Z(t)^2}{2} + \text{correction}\).

To find the correction, we apply \(\frac{d}{dt} \bar{Z}(t) = f(t)\) with \(f(t) = \frac{Z(t)^2}{2}\).

\[\frac{d}{dt} \left( \frac{Z(t)^2}{2} \right) = 2 \frac{Z(t)}{2} \times \frac{d}{dt} t = \frac{Z(t)}{2} = \int_0^t Z(t) \, dt + \frac{1}{2} t \]

Then, \(\int_0^t \bar{Z}(t) \, dt = \frac{Z(t)^2}{2} - \frac{t}{2}\).

(c) 
\[E(\bar{Z}(t)^2) = \text{Var}(\bar{Z}(t)) = t, \quad \text{since } \bar{Z}(t) \sim \mathcal{N}(0, t).\]

(d) 
\[P(\bar{Z}(t) > q) = P\left( \frac{\bar{Z}(t)}{\sqrt{t}} > \frac{q}{\sqrt{t}} \right) = P\left( Z > \frac{q}{\sqrt{t}} \right) = 1 - \Phi\left( \frac{q}{\sqrt{t}} \right) \]

\(\bar{Z}(t) \sim \mathcal{N}(0, t)\)

(e) 
We saw in class that \(E(X(t)) = X(t)\).

(f) 
\[\text{Var}(\bar{Z}(t) | \bar{Z}_s) = \text{Var}(\bar{Z}_t - \bar{Z}_s + \bar{Z}_s | \bar{Z}_s) = \text{Var}(\bar{Z}_t - \bar{Z}_s) = \text{Var}(\bar{Z}_t - \bar{Z}_s / \sqrt{2}) = \frac{t}{2} - \frac{s}{2}, \quad \text{given } \bar{Z}_s, \bar{Z}_s \text{-independent.}\]
Problem 9.

(i) \[ d U(t) = d(\mathcal{Z}^2) - 3d\left(\int_{0}^{t} Z_u \, du\right) \]
\[ = \mathbb{E}_t[\mathcal{Z}^2] + \frac{1}{2}\mathbb{E}_t[\mathcal{Z}^2] \, dt - 3\mathbb{E}_t[\mathcal{Z}^2] \, dt = \mathbb{E}_t[\mathcal{Z}^2] + (1-3\mathbb{E}_t[\mathcal{Z}^2]) \, dt \]

\[ \text{Un Ito's m.f.o.} \]
\[ \text{Term with } f(y) = y^2 \]

(ii) \[ f(y) = e^{3-\frac{y}{2}} \]
\[ d V(t) = d f(\mathcal{Z}, t) = \frac{df}{dt} \, dt + \frac{df}{d\mathcal{Z}} \, d\mathcal{Z} \]
\[ = e^{3-\frac{y}{2}} \left( \frac{1}{2} \, dt + \frac{1}{2} \, d\mathcal{Z} \right) \]
\[ = e^{3-\frac{y}{2}} \, dt + e^{3-\frac{y}{2}} \, d\mathcal{Z} \]
\[ = e^{3-\frac{y}{2}} \, d\mathcal{Z} \Rightarrow \text{a martingale} \]

Problem 10.

\[ \frac{d X(t)}{X(t)} = (r - r_e) \, dt + \sigma \, dZ(t), \Rightarrow \frac{d X(t)}{X(t)} = X(t)(r - r_e) \, dt + X(t) \sigma \, dZ(t) \]

\[ y(t) = \frac{1}{X(t)} = f(X(t)), \text{ with } f(x) = \frac{1}{x}. \]

\[ \frac{df}{dx} = 0, \quad \frac{df}{dx} = -\frac{1}{x}, \quad \frac{df}{dx^2} = \frac{2}{x^2}. \]

By Ito's formula:

\[ d y(t) = \frac{df}{dt} \, dt + \frac{df}{dx} \, dX(t) + \frac{1}{2} \frac{df}{dx^2} \, (dX)^2 \]
\[ = \left( \frac{df}{dx} \right) \left( \frac{1}{X(t)} \right)^2 \, dt + \frac{1}{2} \frac{2}{X(t)^3} \, (dX)^2 \]
\[ = -\frac{1}{X(t)^2} \left[ X(t) \left( (r - r_e) \, dt + \sigma \, dZ(t) \right) \right] + \frac{1}{X(t)^3} \, \sigma^2 \, X(t) \, d t \]
\[ = -\frac{1}{X(t)} \left[ - (r - r_e) + \sigma^2 \right] \, dt - \frac{1}{X(t)} \, \sigma \, dZ(t) \]
\[ = y(t) \left[ \sigma^2 - (r - r_e) \right] \, dt - y(t) \sigma \, dZ(t) \]
Problem 11.

Here, $y(t) = t e^{-2x(t)}$.

Thus, $f(t, x) = t e^{-2x}$.

\[
\begin{align*}
\frac{\partial}{\partial t} e^{-2x} &= e^{-2x} \quad \frac{\partial}{\partial x} e^{-2x} = -2t e^{-2x} \\
\frac{\partial}{\partial x} &= 4t e^{-2x}
\end{align*}
\]

\[
\begin{align*}
dy(t) &= e^{-2x} \, dt - 2te^{-2x} \, dx(t) + \frac{1}{2} \left( 4 + e^{-2x} \right) \, z^2(t) \, dt \\
&= \left( e^{-2x} \, dt - 2te^{-2x} \, dx(t) \right) + \left( 4te^{-2x} \, z^2(t) \, dt \right) \\
&= \left[ e^{-2x} + 2te^{-2x} \, z^2(t) \right] \, dt - \left[ 2te^{-2x} \, z^2(t) \right] \, dt
\end{align*}
\]

\[
\begin{align*}
\frac{dy(t)}{y(t)} &= \left[ \frac{1}{t} - 2z^2(t) \right] \, dt - 2z^2(t) \, dt \\
\frac{a(t, z)}{y(t)} &= \frac{1}{y(t)} + 2z^2(t) \\
a(t, z) &= \frac{1}{y} + 2z^2 \\
a(2, z) &= \frac{1}{y} + 2(\frac{1}{2}) \\
&= 1 \\
b(t, z) &= -2z \\
b(2, z) &= -2(\frac{1}{2}) \\
&= -1
\end{align*}
\]
Problem 12.

\[ S(t) = S(0) \cdot e^{\sigma Z(t) + (\alpha - \frac{1}{2} \sigma^2) t} = 100 \cdot e^{0.3Z(t) + (0.1 - 0.5 \cdot 0.3^2) t} = 100 \cdot e^{0.3Z(t) + 0.055t} \]

(i)

\[ a = 100, \quad b = 0.3, \quad c = 0.055 \]

(ii) Recall that

\[ dS(t) = S(t) (\alpha - \beta) dt + S(t) \sigma d\mathcal{W}(t) \]

\[ = S(t) 0.1 dt + S(t) 0.3 d\mathcal{W}(t) \]

Then,

\[ a(x) = 0.1 x \quad \text{and} \quad \rho(x) = 0.3x \]

(iii) \[ Y(t) = f(S(t)), \quad \text{with} \quad f(x) = x^2, \quad \frac{d\ell}{dt} = 0, \quad \frac{d\ell}{dx} = 2x, \quad \frac{d^2\ell}{dx^2} = 2 \]

By Itô's formula,

\[ dY(t) = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial x} dS(t) + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} (dS(t))^2 \]

\[ = 2S(t) \left[ 0.1S(t) dt + 0.3S(t) d\mathcal{W}(t) \right] + S(t)^2 (0.3)^2 dt \]

\[ = \left[ 0.2 + (0.3)^2 \right] S(t)^2 dt + \left[ 0.6 \right] S(t)^2 d\mathcal{W}(t) \]

Thus,

\[ \mathcal{Y}(t) = 0.29 Y(t) dt + 0.6 \cdot Y(t) d\mathcal{W}(t) \]

\[ \Rightarrow \mathcal{Y}(t) = 0.29 \quad \text{and} \quad \eta(t) = 0.6 \]


Problem 13.

a) 
\[ \frac{d}{dt} D(t) = \frac{d}{dS} (dS(t)) + \frac{1}{2} \frac{d^2}{dS^2} (dS(t))^2 + \frac{d}{dt} \cdot dt \]
\[ = 0.5 \left[ 0.5 dt + 0.4 dZ(t)^2 \right] + 0.05 \left( \frac{1}{2} \right)(0.4)^2 dt + 0.1 dt \]
\[ = 0.25 dt + 0.2 dZ(t) + 0.004 dt + 0.1 dt \]
\[ = 0.354 dt + 0.2 dZ(t) \]

Above we used that Delta is the partial derivative of the premium D with respect to S, Gamma is the second partial derivative of the premium D with respect to S, and theta is the partial derivative of the premium D with respect to t.

b) 
By the Black-Scholes equation (which is valid since we are assuming that S follows a geometric B.M.),
\[ \theta + \frac{1}{2} \sigma^2 S^2 \theta + S(r - d) \Delta - r F = 0. \]

Then,
\[ 0.1 + \frac{1}{2} (0.4)^2 (10) (0.05) + 10 (0.05 - 0) 0.5 - 0.05 F = 0. \]

Thus,
\[ F = \frac{1}{0.05} \left\{ 0.1 + \frac{1}{2} (0.4)^2 (10) (0.05) + 10 (0.05 - 0) 0.5 \right\} \]
\[ = 15.0. \]
Problem 14.

We use the Black-Scholes Equation:

\[
\frac{\partial F}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} + \left(\gamma - \delta \right) S \frac{\partial F}{\partial S} - r F = 0.
\]

Here, \( F(t, S) = S^a e^{0.08 t} \).

Thus,

\[
\frac{\partial F}{\partial t} = S^a e^{0.08 t} (0.08)
\]

\[
\frac{\partial F}{\partial S} = a S^{a-1} e^{0.08 t}
\]

\[
\frac{\partial^2 F}{\partial S^2} = a(a-1) S^{a-2} e^{0.08 t}
\]

Plug into the BS equation,

\[
x^a e^{0.08 t} + \frac{1}{2} \sigma^2 S^2 a(a-1) S^{a-2} e^{0.08 t}
\]

\[
+ (0.03 - 0.06) x a x^{a-1} e^{0.08 t} - 0.03 F = 0.
\]

Recall that \( F = S^a e^{0.08 t} \) and \( \text{Var}(\ln S(t)) = \sigma^2 t = 0.01 t \).

Thus,

\[
0.08 F + 0.01 a(a-1) F + (-0.03) a F - 0.03 F = 0
\]

\[
\Rightarrow \text{Divide by } F \quad 0.08 + \frac{1}{2} 0.01 a(a-1) - 0.03 a - 0.03 = 0
\]

\[
\Rightarrow a^2 - 35 a + 50 = 0
\]

Solving for \( a \),

\( a = 2 \) or \( a = 5 \).

Problem 15.

(17.2) The cost of delta-hedging is \( \Delta \) times the stock price, or \( \Delta S \). The elasticity is \( \frac{\Delta S}{S} = \frac{15}{4} = 3.5 \). So the risk premium of the call, \( \gamma - r \), is 3.5 times the risk premium of the stock. The risk premium of the stock is \( a - r = (0.16 + 0.02) - 0.06 = 0.12 \). It follows that \( \gamma - r = 3.5(a - r) = 0.42 \), and \( \gamma = 0.06 + 0.42 = 0.48 \).
Problem 16.

\[ V(t) = x S_z(t) + 100 S_1(t) + K e^{-rt} \]

\[ \begin{align*}
\Delta V(t) &= x \Delta S_z(t) + 100 \Delta S_1(t) \\
&= x \left[ 0.04 S_z(t) \Delta t + \sigma S_z(t) \Delta Z(1) \right] \\
&+ 100 \left[ 0.03 S_1(t) \Delta t + 0.3 S_1(t) \Delta Z(1) \right] \\
&= \left\{-x \sigma S_z(t) \Delta t + (100)(0.3) S_1(t) \Delta t \right\} \Delta t + \left\{ \text{stuff} \right\} \Delta t
\end{align*} \]

Then, in order for \( V \) to be risk-free,

\[ -x \sigma (60) + 100 (0.3) (40) = 0 \quad - \quad (\text{x}) \]

We only need to determine \( \sigma \).

By the non-arbitrage condition of Sharpe Ratio:

\[ \frac{0.09 - 0.05}{0.3} = \frac{0.04 - 0.05}{-\sigma} \]

So, \( \sigma = 0.15 \). Plugging in (x),

\[ -0.15 (60) x + 100 (0.3) (40) = 0 \]

Thus,

\[ x = \frac{100 (0.3) (40)}{0.15 (60)} = \frac{4000}{3} = 133.33 \ldots \]

Note: The last term in \( V(t) \) should be \( K e^{-r(1-t)} \). This does not affect the rest of the solution.
Problem 17.

By the Black-Scholes equation (19.1),

$$0.5S^2 \sigma^2 C_{ss} + C_t + C_{s} S(r - \delta) = r C$$

Let's differentiate $C$ and substitute into this equation.

$$C_S = \frac{e^{rt}}{S(t)}$$

$$C_{ss} = -\frac{e^{rt}}{S(t)^2}$$

$$C_t = re^{rt} \ln S(t)$$

Therefore

$$-\frac{0.5S(t)^2(0.09)e^{rt}}{S(t)^2} + 0.05e^{rt} \ln S(t) + \frac{e^{rt} S(t)(0.05 - \delta)}{S(t)} = 0.05e^{rt} \ln S(t)$$

Divide through by $e^{rt}$, and cancel out $S(t)$ where it appears in the numerator and denominator of a fraction.

$$-0.5(0.09) + 0.05 \ln S(t) + (0.05 - \delta) = 0.05 \ln S(t)$$

$$-0.045 + 0.05 - \delta = 0$$

$$\delta = \boxed{0.005}$$
Problem 18.

We first figure out the payoff of the option:

\[ \Phi(S_T) = \begin{cases} 
S_T & \text{if } S_T < K \\
0 & \text{if } S_T \geq K 
\end{cases} \]

Here, \( K = 1000 - 0.4 \times 1000 = 600 \) (down by more than 40%).

Next, we express the payoff in terms of cash-or-nothing or asset-or-nothing options. Clearly,

\[ \Phi(S_T) = S_T (1 - I_{S_T \geq K}) = S_T - S_T I_{S_T \geq K}. \]

Then,

\[ F_{0,T}^p(\Phi(S_T)) = F_{0,T}^p(S_T) - \frac{F_{0,T}^p(S_T I_{S_T \geq K})}{\text{Asset-or-nothing option}} \]

\[ = e^{-r_T} S_0 - e^{-r_T} S_0 N(d_1) \]

\[ d_1 = \ln\left(\frac{1000}{600}\right) + (0.025 - 0.02 + \frac{1}{2} \times 0.2^2) \frac{1}{0.2 \sqrt{t}} \approx 2.679 \approx 2.78 \]

\[ N(d_1) = 0.9963. \]

Finally,

\[ F_{0,T}^p(\Phi(S_T)) = e^{-0.02} \times 1000 (1 - 0.9963) = 3.6247. \]

This needs to be multiplied by the 1,000,000 of written options.
Problem 19.

The pay-off can be decomposed as follows:

\[ X = 30 \sum_{S_1 \leq 50} + 20 \sum_{S_0 < S_1 \leq 60} = 30 \sum_{S_1 \leq 50} + 20 \sum_{S_0 < S_1 \leq 60} - 20 \sum_{S_0 < S_1 \leq 60} \]

Therefore,

\[ F_{S_t}(X) = 30 \mathcal{C}^{\text{ch}}(K=50) + 20 \mathcal{C}^{\text{ch}}(K=50) - 20 \mathcal{C}^{\text{ch}}(K=60) \]

\[ \mathcal{C}^{\text{ch}}(K=50) = \mathbb{E}^{\mathcal{R}} N(-d_1) = \mathbb{E}^{\mathcal{R}} \times 0.9801 = 0.9659 \]

\[ d_1 = \ln \left( \frac{50}{50} + (0.03 - 0.5 \times 0.1) \right) = 0.05 \]

\[ N(d_1) = 1 - N(0.05) = 0.9801 \]

\[ \mathcal{C}^{\text{ch}}(K=50) = \mathbb{E}^{\mathcal{R}} N(0.05) = 0.5045 \]

\[ \mathcal{C}^{\text{ch}}(K=60) = \mathbb{E}^{\mathcal{R}} N(0.05) = 0.499 \]

Finally,

\[ F_{S_t}(X) = 30 \mathcal{C}^{\text{ch}}(K=50) + 20 \mathcal{C}^{\text{ch}}(K=50) - 20 \mathcal{C}^{\text{ch}}(K=60) \]

\[ = 30 \times 0.9659 + 20 \times 0.5045 - 20 \times 0.499 = 2.399 \]

Problem 20.

We have \( S_t = 60 \), \( \sigma = 0.25 \), \( r = 0.03 \).

We want to find \( x \) such that

\[ F_{S_t}( (60 - S_t) \mathbb{I}_{S_t \leq 64} ) - F_{S_t}( x \mathbb{I}_{S_t \leq 60} ) = 0 \]

\[ \text{Gap put option with exercise price 60} \]

\[ x \mathbb{I}_{S_t \leq 60} \]

\[ \text{Caché-n-netted cell} \]

\[ d_1 = \ln \left( \frac{60}{60} + (0.03 - 0.5 \times 0.1) \right) = 0.05 \]

\[ d_2 = 0.55 - 0.5 \times 0.15 = -0.15 \]

\[ N(d_1) = 1 - N(0.05) = 0.9708 = 0.2912 \]

\[ N(d_2) = N(0.15) = 0.5996 \]

\[ F_{S_t}( (60 - S_t) \mathbb{I}_{S_t \leq 64} ) = \mathbb{E}^{\mathcal{R}} N(-d_1) = \mathbb{E}^{\mathcal{R}} \times 0.9562 \]

\[ = 0.49253 \]

\[ F_{S_t}( x \mathbb{I}_{S_t \leq 60} ) = \mathbb{E}^{\mathcal{R}} N(-d_1) = \mathbb{E}^{\mathcal{R}} \times 0.9562 \]

\[ = 0.49253 \]

Finally,

\[ x = \frac{F_{S_t}( (60 - S_t) \mathbb{I}_{S_t \leq 64} )}{F_{S_t}( x \mathbb{I}_{S_t \leq 60} )} = \frac{16.5825}{0.9253} = 38.99 \]