Solutions. Practice Problems for Exam 1. STAT 473.

Problem 1.

1. By put-call parity,

\[ 40e^{-r} - Se^{-\delta} = 1.12 - 8.25 = -7.13 \]
\[ 50e^{-r} - Se^{-\delta} = 6.47 - 4.15 = 2.32 \]

Subtracting the first from the second

\[ 10e^{-r} = 7.13 + 2.32 = 9.45 \]

By put-call parity at 45,

\[ 45e^{-r} - Se^{-\delta} = P_2 - 5.40 \]
\[ 5e^{-r} + (40e^{-r} - Se^{-\delta}) = P_2 - 5.40 \]
\[ \frac{9.45}{2} + (-7.13) = P_2 - 5.40 \]
\[ P_2 = \frac{9.45}{2} - 7.13 + 5.40 = \boxed{2.995} \]

Problem 2.

(A) (I) only
(B) (II) only
(C) (III) only
(D) (I) and (II) only
(E) (I) and (III) only

Recalling the no-arbitrage property

\[ 0 \leq c(x, t) - c(x, t) \leq (x_2 - x_1) e^{\gamma t} \]

I is true.

Instead of trying to play with the formulas,

let's rather consider only the payoffs of \( P(45, t) - C(50, t) + S \) in all cases. The payoff is:

\[ (45 - S) + (S - 50) + S \]

If \( S < 45 \),

Payoff = \((45 - S) + S = 45\)

If \( 45 \leq S \leq 50 \),

Payoff = \$0\n
If \( S > 50 \),

Payoff = -(S - 50) + S = 50.

So, \( 45 \leq \text{Payoff} \leq 50 \Rightarrow 45e^{-\gamma t} \leq P(45, t) - C(50, t) + S \leq 50e^{-\gamma t} \).

So, (I) & (III) are true.
Problem 3.

(A) 35 yen
(B) 37 yen
(C) 39 yen
(D) 41 yen
(E) 43 yen

We want

\[ P\left(\frac{1}{0.011}, \frac{1}{0.008}, 4\right). \]

By the formula

\[ C_3(x_0, k, T) = x_0 k P\left(\frac{1}{x_0}, \frac{1}{k}, T\right), \]

\[ P\left(\frac{1}{0.011}, \frac{1}{0.008}, 4\right) = \frac{C_3(0.011, 0.008, 4)}{0.011 \times 0.008} \]

To find \[ C_3(0.011, 0.008, 4), \] we use put-call parity:

\[ C_3 = P_3 + x_0 e^{-r_t T} - K e^{-r_T} \]

\[ = 0.0005 + 0.011 \times e^{-0.015 \times 4} - 0.008 \]

\[ = 0.003764 \]

\[ P\left(\frac{1}{0.011}, \frac{1}{0.008}, 4\right) = \frac{0.003764}{0.011 \times 0.008} = 42.77 \approx 43. \]

Problem 4.

5. By slope, the difference between a 35-strike call option and a 40-strike call option for a European option cannot be greater than 5 discounted at interest. See Quiz 2-6. Therefore the largest difference of price between the calls is \(5e^{-rT} = 5e^{-0.05} = 4.75615.\) This implies that a 40-strike call option must be worth at least \(10 - 4.75615 = 5.24385.\)

By convexity, a 40-strike call option must be worth at most the linearly interpolated value of 35-strike and 45-strike, or \((\frac{3}{5})(10) + (\frac{2}{5})(6) = 6.\) By put-call parity, the put premium is

\[ P(40, 40, 1) = C(40, 40, 1) + 40e^{-0.05} - 40e^{-3.02} = C(40, 40, 1) - 1.15877 \]

Plugging in \(5.24385 \leq C(40, 40, 1) \leq 6,\) we get \(4.08506 \leq P(40, 40, 1) \leq 4.84123.\)
Problem 5.

1. The premiums decrease as the strike price increases, therefore no arbitrage based on that test.

2. The difference in premium never exceeds the difference in strike prices so no arbitrage based on that test.

3. Convexity Test
\[
\frac{P(k_2) - P(k_1)}{k_2 - k_1} \leq \frac{P(k_3) - P(k_2)}{k_3 - k_2}
\]

\[
eq \frac{4.94}{10} \leq \frac{4}{5} \quad \text{so true so no arbitrage}
\]

i true, others false

Problem 6.

\[ P_0 = 0.06, \quad r = 0.03, \quad \sigma^2 = 1.5 \times 1, \quad P^b_w(k = 0.5, T/5) = 0.023, \quad P^b_w(k = 2 \pi, T/5) = 0.5 \]

(a) Compute the arbitrage-free price of \( P^d(k') = \frac{e^{rT}}{k'} C_I \left( \frac{1}{k'} \right) \).

By call/put parity,

\[
C_I \left( \frac{1}{k'} \right) = P^b_w \left( \frac{1}{k'} \right) + \frac{e^{-rT}}{k'} + \frac{e^{-rT}}{k^{1/2}}
\]

\[
= 0.023 + \frac{1}{0.5} e^{-0.03 \left( \frac{1}{5} \right)} - 0.5 e^{-0.03 \left( \frac{1}{2} \right)} = 0.177
\]

Then,

\[ P^d(k) = (1.5) \times 2 \times 0.177 \checkmark = 0.531 \neq 0.5 = P^d_{\text{market}} \]

(b) Since \( P^d_{\text{market}} \leq P^d_{\text{synthetic}} = 0.531 \)

\[ \overline{\text{Buy}} \quad \overline{\text{Sell}} \]

To see what selling \( P^d_{\text{synthetic}} \) means, note that

\[ P^d_{\text{synthetic}}(k') = \frac{e^{rT}}{k'} C_I \left( \frac{1}{k'} \right) \]

\[ \begin{align*}
&= \frac{e^{rT}}{k'} \left[ P^b_w \left( \frac{1}{k'} \right) + \frac{e^{-rT}}{k'} - \frac{1}{k'} e^{-rT} \right] \\
&= \frac{e^{rT}}{k'} \left[ e^{-rT} - \frac{e^{-rT}}{k'} \right]
\end{align*} \]

Transaction at \( t = 0 \)

1. Buy the dollar-denominated put on \( D \)

2. Sell \( K \) units of the pound-denominated put on \( B \)

3. Buy \( k' e^{rT} \) dollars (Sell treasuries in dollars)

4. Sell \( e^{-rT} \) pounds (buy treasuries in pounds)