STAT 473. Exam 1 Solutions.  
Spring 2015. 

Problem 1.

\[
\text{Given: } PV(D_0) = 2 \left( 1 - \frac{1}{1 + 0.04} \right) + 2 \left( 1 + \frac{1}{1 + 0.04} \right) = 3.9536 
\]

\[
C - P = S_0 - PV(D_0) - K e^{rT} \Rightarrow \frac{PV(D_0) + K e^{rT}}{3.9536 + 95 e^{0.05/2}} = 97.07265 
\]

Thus, we need
\[
\frac{10,000 e^{0.05}}{97.07265} = 102.755 \text{ shares (buy)} 
\]

\[
C - P = S_0 - PV(D_0) - K e^{rT} 
\]

\[
4 - 2 = 100 - 3.9536 - 95 e^{0.05/2} = 2.92392 
\]

Since
\[
\frac{C - P < S_0 - PV(D_0) + K e^{rT}}{\text{Buy}} \Rightarrow \text{Sell} \quad \begin{array}{c|c|c}
\text{Transaction at } t=0 & \text{Cashflow} \\
\hline
\text{Buy call} & -4 \\
\text{Sell put} & +2 \\
\text{Sell stock} & +100 \\
\text{Sell money} & 98 \\
\hline
\text{P/L} & 0 \\
\end{array}
\]

Transaction at \( t=T=1 \):
\[
\begin{array}{c|c|c|c|c}
\text{Call} & \text{Put} & \text{Sell stock} & \text{Sell money} & \text{P/L} \\
\hline
S_h > 95 & 0 & 0 & 98 x e^{0.05} & 0.9962 \\
S_h < 95 & -(S_h - 95) & 0 & 98 x e^{0.05} & 0.9962 \\
S_h = 95 & 0 & 0 & 98 x e^{0.05} & 0.9962 \\
\end{array}
\]

Final value of dividends paid at 2 months and 5 months.
Problem 2.

\[ x(0) = 0.85 \text{ } \frac{\$}{\text{€}}. \]
\[ r_f = 2\%. \quad r_e = 4\%. \]
\[ C^e_+ (k = 0.8 \text{ } \frac{\$}{\text{€}}, T = 1) = 0.08. \]

According to Call / Put Parity,
\[ C^e_+ - P^e_+ = x(0) e^{r_e T} - K - e^{-r_f T}. \]

\[ \Rightarrow P^e_+ (k = 0.8 \text{ } \frac{\$}{\text{€}}, T = 1) = C^e_+ - x(0) e^{r_e T} - K - e^{-r_f T} \]
\[ = 0.047487915. \]

\[ P^e_+ (k = 0.8 \text{ } \frac{\$}{\text{€}}, T = 1) = x(0) - K \cdot C^e_+ (k = 1.25 \text{ } \frac{\$}{\text{€}}, T = 1) \]
\[ \Rightarrow C^e_+ (1.25, 1) = \frac{P^e_+ (0.8, 1)}{x(0) - K} \]
\[ = 0.06838517 \text{ } \text{€}. \]
Problem 3.

(a) \( \text{SP}(t) = \text{SP 500 price at time } t, \quad \text{ND}(t) = \text{NASDAQ price at time } t \)

\( \text{SP}(0) = 30, \quad \delta_{\text{SP}} = 0.02, \quad \text{ND}(0) = 75, \quad \delta_{\text{ND}} = 0.05 \)

\( C\left( \text{ND}, 2.5 \text{SP}, T \right) = 2.50 \)

We want \( C\left( \text{SP}, 0.4 \text{ND}, T \right) \).

By call/put parity for exchange options:

\[
C\left( \text{SP}, 0.4 \text{ND}, T \right) - P\left( \text{SP}, 0.4 \text{ND}, T \right) = \text{SP}(0) e^{-\delta_{\text{SP}} T} - 0.4 \text{ND}(0) e^{-\delta_{\text{ND}} T}
\]

Using that

\( P\left( \text{SP}, 0.4 \text{ND}, T \right) = C\left( 0.4 \text{ND}, \text{SP}, T \right) = 0.4 C\left( \text{ND}, 2.5 \text{SP}, T \right) \),

We get

\[
C\left( \text{SP}, 0.4 \text{ND}, T \right) = 0.4 C\left( \text{ND}, 2.5 \text{SP}, T \right) = \text{SP}(0) e^{-\delta_{\text{SP}} T} - 0.4 \text{ND}(0) e^{-\delta_{\text{ND}} T}
\]

Finally,

\[
C\left( \text{SP}, 0.4 \text{ND}, T \right) = (0.4)(2.5) + (30) e^{-0.02} - 0.4 (75) e^{-0.05}
\]

\[
= 1.8691
\]

(b) From the previous part,

\[
C\left( \text{SP}, 0.4 \text{ND}, T \right) = 0.4 C\left( \text{ND}, 2.5 \text{SP}, T \right) = \text{SP}(0) e^{-\delta_{\text{SP}} T} - \text{ND}(0) (0.4) e^{-\delta_{\text{ND}} T}
\]

Then,

\[
\text{ND}(0) e^{-\delta_{\text{ND}} T} = \frac{C\left( \text{SP}, 0.4 \text{ND}, T \right)}{0.4} - \text{SP}(0) e^{-\delta_{\text{SP}} T} + 0.4 C\left( \text{ND}, 2.5 \text{SP}, T \right)
\]

1 share = 1

\( \text{ND} \) at time \( T \)

\# of shares of

\text{SP needed today}

\[
\eta = \frac{e^{-0.02}}{0.4} = 2.4505
\]
Problem 4.

5. By slope, the difference between a 35-strike call option and a 40-strike call option for a European option cannot be greater than 5 discounted at interest. See Quiz 2-6. Therefore the largest difference of price between the calls is $5e^{-rT} = 5e^{-0.05} = 4.75615$. This implies that a 40-strike call option must be worth at least $10 - 4.75615 = 5.24385$.

By convexity, a 40-strike call option must be worth at most the linearly interpolated value of 35-strike and 45-strike, or $(\frac{5}{10})(10) + (\frac{5}{2})(6) = 6$. By put-call parity, the put premium is

$$P(40,40,1) = C(40,40,1) + 40e^{-0.05} - 40e^{-0.02} = C(40,40,1) - 1.15877$$

Plugging in $5.24385 \leq C(40,40,1) \leq 6$, we get $4.08508 \leq P(40,40,1) \leq 4.84123$

Problem 5.

\[ S_0 = 100, \quad r = 5\% \]
\[
C\left(K_1 = 100e^{r_1} = 105.127, T_1 = 1\right) = 11.924,
\]
\[
C\left(K_2 = 100e^{2r} = 107.788, T_2 = 1.5\right) = 11.50
\]
\[
C_{EU}(K_1, T_1) \leq C_{EU}(K_2 = K_1e^{(T_2-T_1)}, T_2)
\]

Since
\[
C_{EU}(K_1, T_1) > C_{EU}(K_2 = K_1e^{(T_2-T_1)}, T_2)
\]

Sell High & Buy Low

Strategy:
Sell the first Call with shorter expiration
Buy the second Call with longer expiration.
Invest the difference: $11.924 - 11.50 = 0.424$.

At $t = 1$, $S_1 = 110$

The first call is exercised against us:
We get $K_1$, and need a share of stock.
Therefore, we short sell the stock, and invest $K_1$

At $t = 1.5$, $S_{1.5} = 110$

We exercise the second call.
We paid $K_2$, in exchange for a share of stock.
We then use this share to close our short-sell position happened at $t = 1$.

We close all the positions and collect the interests on investments.

Total Profit = $0.424 \cdot e^{r_2} + K_1 \cdot e^{(T_2-T_1)} - K_2 = 0.457023$