1. Suppose that \( \{Z_t\} \) is a purely random process with mean zero and variance \( \sigma_Z^2 \). Consider the following first order MA processes:

\[
A : \ X_t = Z_t + \theta Z_{t-1} \\
B : \ X_t = Z_t + \frac{1}{\theta} Z_{t-1}
\]

Show that the processes \( A \) and \( B \) have exactly the same ac.f.

2. For the autocovariance function, \( \gamma(t_1, t_2) = E[\{X_{t_1} - \mu(t_1)\}\{X_{t_2} - \mu(t_2)\}] \), show that

\[
\gamma(t_1, t_2) = E\{X_{t_1}X_{t_2} - \mu(t_1)\mu(t_2)\}
\]

3. Following the R code below to generate records of length 500 of AR(1) with \( \alpha = 0.5 \) and MA(1) with \( \beta_1 = 0.5 \), and compute their ac.f. Give the two plots of the ac.f. Does the ac.f. of MA(1) display its benchmark property of “cut off”? If so, what is the “cut off” time point?

```r
ar1 <- arima.sim(n=500, list(ar=0.5))
acf(ar1)

ma1 <- arima.sim(n=500, list(ma=0.5))
acf(ma1)
```

4. Exercise 3.1

5. Exercise 3.4. You can just tabulate \( \rho(k) \) instead of plotting them.