READING - Montgomery - Chapter 3

1. Montgomery 3.15.


3. The following data are the treatment means from an experiment where each treatment was randomly and equally allocated to a total of 27 experimental units.

<table>
<thead>
<tr>
<th>Trt</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Suppose the experimenter planned to test the following four hypotheses and assume $MS_E = 5$.

- a) $H_0: 3\mu_2 = \mu_3 + 2\mu_1$
- b) $H_0: \mu_1 = \mu_3$
- c) $H_0: \mu_1 + \mu_2 + \mu_3 = 9$
- d) $H_0: \mu_1 + \mu_3 = 2\mu_2$

   a) Using $\alpha = .05$, test each of the four hypotheses (two sided).
   b) Which of the four linear combinations of means are contrasts? Why?
   c) Are any pairs of contrasts orthogonal? Which ones and why?
   d) Suppose the experimenter was also interested in testing $H_0: \mu_1 = 0$. Since the sample mean of the third group is zero, it appears this is the same test as Hypothesis b). Is it? Explain why or why not?

4. Suppose you performed an ANOVA with $a = 4$ treatments and $n = 6$ observations per treatment. If the $MS_E = 36$ and $\alpha = .05$, what would the minimum difference have to be between any two means for you to conclude they were significantly different if

   A You performed the LSD comparison procedure?
   B You performed Tukey’s multiple comparison procedure (use Table VII)?
   C You performed Bonferroni’s multiple comparison procedure?
   D You performed Scheffe’s procedure?
   E Explain the relationship between power and the minimum difference. Also state which of the four is the most powerful and which is the least powerful comparison procedure.

5. A psychologist has tested 20 independent hypotheses. The 20 unadjusted p-values she obtained are as follows: 0.0045, 0.39, 0.0015, 0.87, 0.0020, 0.14, 0.0006, 0.04, 0.15, 0.005, 0.02, 0.31, 0.06, 0.86, 0.62, 0.01, 0.00003, 0.0373, 0.46, and 0.08. Which, if any, hypotheses can she reject controlling the FDR at 0.05? Which, if any, hypotheses can she reject controlling the experimentwise error rate at 0.05?
6. Ten needles were randomly selected from a branch of a loblolly pine tree. The stomata (microscopic breathing holes) are arranged in rows. On each needle, four rows are randomly selected and the number of stomata per centimeter for each of the rows was determined. The data below is in the file named **stomata.dat**.

<table>
<thead>
<tr>
<th>Needle</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>149</td>
<td>136</td>
<td>143</td>
<td>121</td>
<td>148</td>
<td>129</td>
<td>127</td>
<td>134</td>
<td>117</td>
<td>129</td>
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<tr>
<td></td>
<td>143</td>
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<td>142</td>
<td>133</td>
<td>121</td>
<td>134</td>
<td>130</td>
<td>137</td>
<td>128</td>
<td>132</td>
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<tr>
<td></td>
<td>138</td>
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<td>124</td>
<td>126</td>
<td>124</td>
<td>127</td>
<td>123</td>
<td>119</td>
<td>117</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td>131</td>
<td>143</td>
<td>134</td>
<td>130</td>
<td>128</td>
<td>113</td>
<td>125</td>
<td>130</td>
<td>118</td>
<td>137</td>
</tr>
</tbody>
</table>

**a** Why is the random effects model appropriate here?

**b** Estimate all relevant variance components.

**c** What percentage of the overall variation in stomata number per centimeter is due to the needle?

**d** Construct a 95% CI for this ratio.

**e** Compute a 95% confidence interval for the average number of stomata per centimeter.

7. A sociologist is interested in studying the ability of teachers from low income areas of major cities to cope with stress. Six schools were randomly chosen from low income areas and from each of these schools, five teachers were randomly chosen. The following table summarizes the average coping score (higher the score, the better the ability to cope) for each of these schools.

<table>
<thead>
<tr>
<th>School</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>92</td>
<td>99</td>
<td>94</td>
<td>109</td>
<td>98</td>
<td>103</td>
<td></td>
</tr>
</tbody>
</table>

**a** If MS_E=36, is there significant variability in average coping scores among schools in low income areas (use \(\alpha = .05\))?

**b** Estimate all variance components.

**c** Suppose the national average coping score for teachers is 105. Test to see if the data support the hypothesis that the average coping score of these teachers is lower than the national average (\(\alpha = .05\)).