Problem 1

(a) Again, let's check the ANOVA table.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>3</td>
<td>489740.1875</td>
<td>163246.7292</td>
<td>12.73</td>
<td>0.0005</td>
</tr>
<tr>
<td>Error</td>
<td>12</td>
<td>153908.2500</td>
<td>12825.6875</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>15</td>
<td>643648.4375</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since the p-value is very small (= 0.0005), I conclude that mixing techniques affect the strength of the cement.

(b) The SAS outputs for the four methods are listed below.

**t Tests (LSD) for tensile**

NOTE: This test controls the Type I comparisonwise error rate, not the experimentwise error rate.

- Alpha: 0.05
- Error Degrees of Freedom: 12
- Error Mean Square: 12825.69
- Critical Value of t: 2.17881
- Least Significant Difference: 174.48

Means with the same letter are not significantly different.

<table>
<thead>
<tr>
<th>t Grouping</th>
<th>Mean</th>
<th>N</th>
<th>mixing</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3156.25</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>2971.00</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>2933.75</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>2666.25</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

**Tukey’s Studentized Range (HSD) Test for tensile**

NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

- Alpha: 0.05
- Error Degrees of Freedom: 12
- Error Mean Square: 12825.69
- Critical Value of Studentized Range: 4.19852
- Minimum Significant Difference: 237.74
Means with the same letter are not significantly different.

Tukey Grouping  Mean  N  mixing  
A  3156.25  4  2  
A  2971.00  4  1  
A  2933.75  4  3  
B  2666.25  4  4  

Bonferroni (Dunn) t Tests for tensile

NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha  0.05  
Error Degrees of Freedom  12  
Error Mean Square  12825.69  
Critical Value of t  3.15268  
Minimum Significant Difference  252.47  

Means with the same letter are not significantly different.

Bon Grouping  Mean  N  mixing  
A  3156.25  4  2  
A  2971.00  4  1  
A  2933.75  4  3  
B  2666.25  4  4  

Scheffe’s Test for tensile

NOTE: This test controls the Type I experimentwise error rate.

Alpha  0.05  
Error Degrees of Freedom  12  
Error Mean Square  12825.69  
Critical Value of F  3.49029  
Minimum Significant Difference  259.13  

Means with the same letter are not significantly different.

Scheffe Grouping  Mean  N  mixing  
A  3156.25  4  2  
A  2971.00  4  1  
A  2933.75  4  3  
B  2666.25  4  4  
The critical differences for the four methods are

<table>
<thead>
<tr>
<th>Method</th>
<th>LSD</th>
<th>Tukey</th>
<th>Bonferroni</th>
<th>Scheffe</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>174.48</td>
<td>237.74</td>
<td>252.47</td>
<td>259.13</td>
</tr>
</tbody>
</table>

(c) The grouping results for each method are listed in the following table, where the treatments within one group are considered as not having significantly different effects and the treatments in different groups are considered as having significantly different effects.

<table>
<thead>
<tr>
<th>Method</th>
<th>Grouping Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSD</td>
<td>{2}, {1, 3}, {4}</td>
</tr>
<tr>
<td>Tukey</td>
<td>{2, 1, 3}, {4}</td>
</tr>
<tr>
<td>Bonferroni</td>
<td>{2, 1, 3}, {4}</td>
</tr>
<tr>
<td>Scheffe</td>
<td>{2, 1, 3}, {4}</td>
</tr>
</tbody>
</table>

The smaller the critical difference is, the more power the method is. The bigger the critical difference is, the more conservative the method is. Hence, among the four methods, LSD is the most powerful one and Scheffe’s method is the most conservative one. However, LSD does not control the over error rate, the other three control the over error rate. Apparently, Tukey’s method should be preferred.

Problem 2

(a) From the result of Problem 1, Assignment 3, $MS_E = 0.1325$ with $DF = 60$. This is a balanced ANOVA design with $a = 5$ and $n = 13$. We have $m = 4$ mean effect differences, so the critical distance for the Bonferroni method is

\[
t_{a/2m}(N - a)\sqrt{2MS_E/n} = t_{0.05/8}(65 - 5)\sqrt{2 * 0.1325/13} = 0.368.
\]

Also, the control mean is 11.54 and the treatment means are 11.00, 11.42, 11.44, 11.28 respectively. Hence the testing results for the Bonferroni method are

<table>
<thead>
<tr>
<th>Pesticide</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Diff</td>
<td>0.54</td>
<td>0.12</td>
<td>0.10</td>
<td>0.26</td>
</tr>
<tr>
<td>Significance</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

(b) From Table IX in Montgomery, $d_{a/2}(4, 60) = 2.51$. So the critical distance for the Dunnet method is

\[
d_a(a - 1, N - a)\sqrt{2MS_E/n} = d_{0.05}(4, 60)\sqrt{2 * 0.1325/13} = 0.358.
\]

Hence the testing results for the Dunnet method is

<table>
<thead>
<tr>
<th>Pesticide</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Diff</td>
<td>0.54</td>
<td>0.12</td>
<td>0.10</td>
<td>0.26</td>
</tr>
<tr>
<td>Significance</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

(c) The significance test results for the treatment-control differences are the same in (a) and (b). In our test, the Bonferroni method is slightly more conservative than the Dunnet method, whereas the latter has more power.