1. Let $X$ = # wins ~ Binomial $(n=25, \ p=\frac{9}{19})$

You have a net gain if and only if $X \geq 13$

Normal approximation:

$P(\text{net gain}) = P(X \geq 13) = P(X \geq 12.5)$

$= P \left\{ \frac{X - np}{\sqrt{npq}} \geq \frac{12.5 - np}{\sqrt{npq}} \right\}
= P \left\{ \frac{X - 11.84}{\sqrt{3.4065}} \geq \frac{12.5 - 11.84}{\sqrt{3.4065}} \right\}
= 0.3956$ (approx)

You can also do a normal approximation based on

$S = Y_1 + Y_2 + \ldots + Y_{25}$ where

net gain on $i$th bet $= Y_i = \begin{cases} +1 & \text{prob.} \ \frac{9}{19} \\ -1 & \text{prob.} \ \frac{10}{19} \end{cases}$

so $\mathbb{E}Y_i = \frac{1}{19}$, $\mathbb{E}Y_i^2 = 1$ (since $Y_i^2 \equiv 1$)

and $\text{var}(Y_i) = \mathbb{E}(Y_i^2) - (\mathbb{E}Y_i)^2 = 1 - \left(\frac{1}{19}\right)^2 = \frac{350}{361}$

But, note that the possible values of $S$ are the odd integers $-25, -23, \ldots, -3, -1, +1, +3, \ldots, +25$

so don't use $\pm \frac{1}{2}$ continuity correction.

$P \left\{ S > 0 \right\} = P \left\{ \frac{S - (-\frac{25}{19})}{\sqrt{\frac{350}{361}}} > 0 - (-\frac{25}{19}) \right\}$

$\approx P \left\{ Z > \frac{25(19)}{4.993} \right\} = \frac{25}{19} \text{ number as before.}$
2. With $B = \text{"biased"}$, $F = \text{"fair"}$

\[
P(B|HH) = \frac{P(B|HH)P(HH)}{P(B|HH)P(HH) + P(F|HH)P(HH)}
\]

\[
= \frac{\frac{3}{4} \times 1}{\frac{3}{4} \times 1 + \frac{1}{4} \times 1}
\]

\[= \frac{3}{4} \times 1 + \frac{1}{4} \times 1 = 1
\]

8. For the result of Part 6:

After seeing $HH$, $H_H$, the chance is $\frac{2}{3}$ that the coin is fair.

so the chance of $H_H$ on the next toss is $\frac{2}{3} \times 1 + \frac{1}{3} \times 2 = \frac{5}{3}$.
### (3) Possible X values

<table>
<thead>
<tr>
<th>X values</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
</table>

### (3b) X-Y values and probabilities

<table>
<thead>
<tr>
<th>X values</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>probability</td>
<td>$\frac{27}{64}$</td>
<td>$\frac{27}{64}$</td>
<td>$\frac{9}{64}$</td>
<td>$\frac{1}{64}$</td>
</tr>
</tbody>
</table>

### (4) X values Y

<table>
<thead>
<tr>
<th>X values</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X = k</td>
<td>Y = 1)</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{12}{27}$</td>
<td>$\frac{6}{27}$</td>
</tr>
</tbody>
</table>

Note that probabilities must sum to 1 in both (3a) and (3b).

### (4a) Detected misprints are a Poisson scatter with intensity 0.9 per page (see box on page 232 [7] Pitman) so the total # of misprints detected X ~ Poisson ($\mu = 200 \times 0.9 = 180$).

#### Normal Approx ($\mu 224$):

\[ P( X > 190 ) = P \left\{ \frac{X - 180}{180} > \frac{189.5 - 180}{180} \right\} \]

\[ \cong P \{ Z > 0.708 \} \cong P \{ Z > 0.71 \} \]

\[ = 1 - 0.7611 = 0.2389 \]
(b) Undetected misprints are a Poisson scatter with intensity 0.1 per page. Let $Y_i = \#$ undetected on page $i$.

\[ P(Y_i \leq 2) = P(Y_i = 0) + P(Y_i = 1) = e^{-0.1} + 0.1e^{-0.1} = 0.9953212 \]

\[ P(\text{no page with } Y_i \geq 2) = P(\text{all } Y_i < 2) = (0.9953212)^{200} = 0.3914 \]

\[ P(\text{some page has } \geq 2 \text{ undetected}) = 1 - 0.3914 = 0.6086 \]

5. Let $I_i = \text{ indicator that penny Head's on page } i$.

\[ X = 1 + 5I_1 + 10I_2 + 25I_3 \]

\[ \text{var } X = \frac{1}{4} + 5^2 \frac{1}{2} + 10^2 \frac{1}{4} + 25^2 \frac{1}{4} = \frac{155}{4} = 38.75 \text{ (since each } I_i \text{ are independent Bernoulli } (\frac{1}{2}), \text{ var is } \frac{1}{4}) \]

6. This problem is a setting due for the Method of Indicators.

Solution 1:
\[ X = I_1 + \ldots + I_9 + I_7 \]

where $I_1 = \text{ indicator that 1st woman surrounded by other women}$

\[ I_7 = \text{ indicator that 6th man surrounded by other women} \]

For any person, that person's neighbors are a random pair from the 15 other people, so

\[ E I_k = P(\text{neighbors both women}) = \frac{\binom{3}{2}}{\binom{15}{2}} = \frac{3}{15} = \frac{1}{5} \]

\[ E I_k = P(\text{neighbors both men}) = \frac{\binom{12}{2}}{\binom{15}{2}} = \frac{12}{35} \]

so

\[ EX = 9 \cdot \frac{1}{15} + 7 \cdot \frac{3}{35} = \frac{12}{5} + \frac{12}{5} = \frac{24}{5} = 4.8 \]
Solution 2, inspired by Heidi Bollinger / Sollee

\[ X = I_1 + \ldots + I_{16} \]

where

\[ I_k = \text{indicator that k\textsuperscript{th} spot on the table is surrounded by women.} \]

The neighbors of spot \( k \) are a random pair from the 16 people, so

\[ E I_k = \frac{\binom{9}{2}}{16} = \frac{9 \cdot 8}{16 \cdot 15} = \frac{3}{10} \]

\[ E X = 16 \times \frac{3}{10} = 4.8 \]

Solution 3, by Sarah Sollee / wife of T. Sollee

\[ X = I_1 + \ldots + I_9 \]

where \( I_k \) = indicator that the spot 2 seats clockwise from woman \( k \) 's seat is occupied by a woman.

\( \left( \text{So } I_k \text{ spot surrounded by 2 females with } I_k \text{ counter-clockwise} \right) \)

\[ E I_k = \frac{8}{15} \quad \text{so } \quad E X = 9 \times \frac{8}{15} = \frac{24}{5} = 4.8 \]

\[ P(X = Y) = \sum_{k=1}^{16} P(X = Y | k) = \sum_{k=1}^{16} P(X = k) P(Y = k) \]

\[ = \sum_{k=1}^{16} \left( \frac{2}{15} \right) \cdot \left( \frac{5}{12} \right) \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{12} \sum_{j=0}^{10} \left( \frac{5}{12} \right) \]

\[ = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{12} \cdot 8 = \frac{1}{7} \]

Or use craps principle:

Simultaneously toss coin and roll die until you can tell whether \( X = Y \) happen.

On each toss-roll, there are 12 possible outcomes, (prob \( \frac{1}{12} \) each):

<table>
<thead>
<tr>
<th>H</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>X \leq Y</td>
<td>STOP</td>
<td>STOP</td>
<td>STOP</td>
<td>STOP</td>
<td>STOP</td>
<td>STOP</td>
</tr>
<tr>
<td>X &gt; Y</td>
<td>Continue</td>
<td>Continue</td>
<td>Continue</td>
<td>Continue</td>
<td>Continue</td>
<td>Continue</td>
</tr>
</tbody>
</table>

Of the 7 outcomes which determine whether \( X = Y \) or \( \neq \), only one has \( X = Y \), so the probability that you stop with the \( X = Y \) outcome is \( \frac{1}{7} \)