1. SHOW YOUR WORK. I may not give credit for a correct answer if I can’t see how you got it.

2. Give answers either as fractions or as decimal numbers to at least three significant digits.

3. Write neatly and clearly. Remember, good penmanship is the key to success in life.

4. There is a normal table at the end of this exam. There is also a fact sheet.

5. If you need more space, use the back of the preceding page. Write “See back of preceding page” in the answer space.

6. Each of the 15 parts is worth 10 points.

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1. Brand $A$ lightbulbs have exponential lifetimes, with mean 1 year. Brand $B$ lightbulbs have exponential lifetimes, with mean 2 years. Brand $C$ lightbulbs have exponential lifetimes, with mean 3 years. I buy one of each and screw them into sockets. Let $T$ be the number still working after 2 years.

(a) $P(T = 3) =$?

(b) $\text{var} (T) =$?
2. A box contains three tickets, labeled 1, 2 and 3. Draw a ticket at random, and let 
$X$ equal the number on the ticket you draw. Without replacing the first ticket, draw 
another ticket from the box, and let $Y$ equal the number on this second ticket. 
(a) Calculate $E(X|Y = 3)$. 

(b) Calculate the correlation $\rho$ between $X$ and $Y$. 
3. Suppose that the heights of male Purdue students are normally distributed, with mean 70 inches and standard deviation 3 inches. (Heights are measured exactly, not to the nearest inch.)

   (a) If you randomly sample Purdue men one-by-one until you get one taller than 75 inches, what is the expected number of men you’ll need to sample?

   (b) If you randomly sample 100 male Purdue students, what is the probability that 30 or more will be taller than 72 inches? (This has nothing to do with part (a).)
(c) What is the probability that the average height of the 100 men in your random sample in (b) will be between 71.5 inches and 72.5 inches?
4. I toss a thumb tack 10 times. The tack lands point-up on 4 tosses and point-down on 6 tosses. Given this information, what is the probability that the first three tosses were all point-down?
5. Magician Mandrake the Mediocre has two white dice. One is a standard, fair, six-sided
die. The other has one side labeled 1, two sides labeled 2, and three sides labeled 3.
Mandrake randomly puts one die into box $A$ and one die into box $B$. He shakes box
$A$ and then lets you look down into the box. You see only the top side of the die.
This shaking of box $A$ and looking into the top is repeated another two times. Suppose
that you see the numbers 1, 2, and 3 on your successive looks.
Assuming that Mandrake is not engaging in any trickery, what is the probability that
box $A$ contains the funny die?
6. Pick a point at random according to the uniform distribution on the interior of a unit square. (A unit square has sides of length 1). What is the median distance between the center of the square and your chosen point?
7. Pick an random married couple. Let $X$ be the wife’s height in inches, and let $Y$ be the husband’s height in inches. Then

\[
\mu_X = 63\text{inches}, \sigma_X = 2.5\text{inches} \\
\mu_Y = 68\text{inches}, \sigma_Y = 2.7\text{inches} \\
\rho = 0.25
\]

Assume that $X + Y$ are bivariate normal.

*(a) What fraction of wives are taller than their husbands?*

* Based on a study by Pearson and Lee of about 1,000 families in England early this century.
(b) Of wives who are 68 inches tall, what fraction have husbands who are at least 72 inches tall?
8. Let $X$ be a random variable with density

$$f(x) = \begin{cases} 
  x & \text{if } 0 < x \leq 1 \\
  2 - x & \text{if } 1 \leq x < 2 \\
  0 & \text{otherwise}
\end{cases}$$

Let $Y = X^2$. Find the density of $Y$. 
9. Let $X$ and $Y$ be independent, exponential random variables with means $EX = EY = 1$. Find $P\{2X > Y + 1\}$
10. Let $X$ and $Y$ have joint density

$$f(x, y) = \begin{cases} 
xy & \text{if } 0 < x < 2, 0 < y < 1 \\
0 & \text{else.}
\end{cases}$$

Let $T = X + Y$ and $W = X - Y$. Find the joint density of $T$ and $W$. 