1. **SHOW YOUR WORK.** I may not give credit for a correct answer if I can’t see how you got it.

2. Give answers either as fractions or as decimal numbers to at least three significant digits.

3. Write neatly and clearly. Remember, good penmanship is the key to success in life.

4. There is a normal table at the end of this exam.

5. If you need more space, use the back of the preceding page. Write “See back of preceding page” in the answer space.

6. Each of the 10 parts is worth 10 points, unless otherwise indicated.

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**Facts**

1. If $X$ is Geometric with success probability $p$, then $EX = \frac{1}{p}$ and $\text{Var } X = \frac{q}{p^2}$.

2. Poisson ($\mu$) distribution:

   \[ P\{X = k\} = e^{-\mu} \frac{\mu^k}{k!} \text{ for } k = 0, 1, 2, \ldots \]

   \[ E(X) = \mu, \text{ SD}(X) = \sqrt{\mu} \]

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△
1. A prisoner of war decides to try to escape over the prison camp fence. If he tries the north fence, his probabilities for successful escape, being captured alive, and getting killed are \( \frac{5}{12}, \frac{1}{3}, \) and \( \frac{1}{4}, \) respectively. For the south fence, the probabilities are \( \frac{1}{4} \) for success, \( \frac{5}{8} \) for capture, and \( \frac{1}{8} \) for death. For the west fence, the probabilities are \( \frac{3}{4} \) for success, 0 for capture, and \( \frac{1}{4} \) for death. The east fence is not a possible escape direction, since the guards live across the east fence.

The prisoner will choose his escape direction by rolling a fair die. If the die comes up 1, 2, or 3, he tries the north fence. He tries the south fence if the die comes up 4 or 5, and he tries the west fence if the die comes up 6.

a.) What is the probability that his escape attempt is successful?

b.) The next day, the other prisoners learn that he succeeded in escaping. Given that he succeeded, what is the conditional probability that he escaped over the north fence?
2. A fair, six-sided die has three sides labeled 3, two sides labeled 2, and one side labeled 1. The die is rolled and $X$ is the result. A fair coin is then tossed $X$ times. Let $Y$ be the number of heads obtained in the $X$ tosses.

a.) (15 pts) Find the joint distribution table for $X$ and $Y$.

b.) (5 pts) Calculate $P(X = 3|Y = 0)$. 


3. The number of deaths in the USA caused by falling in bath tubs has held steady at about 1,000 per year for the last 10 years. *Give an intelligent estimate for the number of days this year on which there is exactly one death by bathtub fall. Explain what assumptions you are making. Also estimate the probability that your guess will be exactly right.

* Confession, I made this up, but pretend it’s true.
4. A fair, five-sided die has sides labeled 1, 2, 3, 4, and 5. If you roll it 100 times, what is the probability that the sum of the 100 results is between 290 and 320, inclusive?
5. Flip a fair coin repeatedly until you get a heads. Let $X$ be the number of flips needed. Then roll a fair die until you get an ace. Let $Y$ be the number of rolls needed.
   a.) Calculate $P(X = 1|X + Y = 4)$.

b.) Calculate $P\{3X > Y\}$. 
6 Deal 5 cards from a well-shuffled standard 52-card deck. Given that there is at least one spade among the 5 cards, what is the conditional probability that there is also at least one club?
7. A computer is programmed to generate a sequence of 250 random numbers. Each random number is randomly chose to be of the 10,000 integers between 0000 and 9999, inclusive, and different random numbers are independent. *For 1 ≤ n ≤ 250, let X be the n\textsuperscript{th} random number. Calculate

\[ P\left(3 = \sum_{n=1}^{250} I\{X_n < n\}\right) \]

where \(I\{X_n < n\}\) equals one if \(X_n < n\) and equals zero otherwise.

* So, you can think of each random number as four independent random digits, each digit equally likely to be 0, 1, \ldots, 9.