Inference on a Single or Multiple Contrasts

Single Contrast:

A contrast is a linear combination of treatment effects, hence can be written as $\sum_i c_i \tau_i$, where the sum of the coefficients $c_i$s must be 0. For this reason, a contrast can be written as $\sum_i c_i \mu_i$.

For the one-way anova model, the confidence interval for a single contrast

$\text{estimate} \pm t_{n,v-\alpha/2} \times (\text{std error of estimate})$,

(Note that $n$ is the total number of observations and $v$ is the number of treatments)

Both the estimate and the standard error are given by SAS. However, the confidence interval for a single contrast is not given by SAS.

A Set of Contrasts: When we have a set of contrasts and need to construct a confidence interval for each of the contrasts, we want all intervals to be simultaneously correct with a certain probability. These intervals will then be called simultaneous 100$(1 - \alpha)$% confidence intervals. Several methods exist and the choice of method depends on the problems. However, each method provides simultaneous confidence intervals of the common form:

$\text{estimate} \pm w \times (\text{std error of estimate})$,

where $w$ varies with each method. The following table summarizes these $w$ values and where to find them.

<table>
<thead>
<tr>
<th>Method</th>
<th>Bonferroni</th>
<th>Scheffe</th>
<th>Tukey</th>
<th>Dunnett</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>$t_{n-v,\alpha/(2m)}$</td>
<td>$\sqrt{\frac{(v - 1)F_{y-1,n-v,\alpha}}{\alpha}}$</td>
<td>$q_{v,n-v,\alpha}/\sqrt{2}$</td>
<td>needs the multivariate t distribution</td>
</tr>
<tr>
<td>Table</td>
<td>A.4, p704</td>
<td>F value in A.6,p706</td>
<td>q value in A.8, p718</td>
<td>A.9, p720</td>
</tr>
</tbody>
</table>

Note:
1. \( m \) is the number of contrasts that we are interested.
2. Bonferroni’s method can be used for all preplanned contrasts.
3. Scheffe’s method can be used for any contrasts.
4. Tukey’s method is only for pair-wise comparisons and is the best method for all pairwise comparisons.
5. Dunnett’s method is only used for treatment-versus-control comparisons.

**Example.** Using data in Ex 4 on page 98, let us do the following problems:

1. Find a 95% confidence interval for the contrast for \( \frac{\tau_1 + \tau_2 + \tau_3}{3} - \frac{\tau_4 + \tau_5 + \tau_6}{3} \).
2. Test the hypothesis that 
   \[
   H_0: \frac{\tau_1 + \tau_2 + \tau_3}{3} - \frac{\tau_4 + \tau_5 + \tau_6}{3} = 0
   \]
   against
   \[
   H_1: \frac{\tau_1 + \tau_2 + \tau_3}{3} - \frac{\tau_4 + \tau_5 + \tau_6}{3} \neq 0.
   \]
3. Find a set of simultaneous 90% confidence intervals for the following preplanned contrasts 
   \( \tau_1 - \tau_4, \tau_2 - \tau_5, \tau_3 - \tau_6 \) and 
   \[
   \frac{\tau_1 + \tau_2 + \tau_3}{3} - \frac{\tau_4 + \tau_5 + \tau_6}{3}
   \]
   using an appropriate method.

Here, the possible choices are Bonferroni and Scheffe. I will illustrate both methods.

*Bonferroni method:*

*Scheffe method:*
*reaction time experiment; chapter 4;

OPTIONS LINESIZE=85;
DATA REACT;
INPUT ORDER TC CUE ELAPTM REACTM;
LINES;
 1 6 2 15 0.256
 2 6 2 15 0.281
 3 2 1 10 0.167
 4 6 2 15 0.258
 5 2 1 10 0.182
 6 5 2 10 0.283
 7 4 2 5 0.257
 8 5 2 10 0.235
 9 1 1 5 0.204
10 1 1 5 0.170
11 5 2 10 0.260
12 2 1 10 0.187
13 3 1 15 0.202
14 4 2 5 0.279
15 4 2 5 0.269
16 3 1 15 0.198
17 3 1 15 0.236
18 1 1 5 0.181
;
PROC GLM DATA=REACT;
CLASS TC;
MODEL REACTM=TC;
ESTIMATE 'AUDITORY-VISUAL' TC 1 1 1 -1 -1 -1 /DIVISOR=3;
ESTIMATE 'AUDITORY-VISUAL, 5 SEC' TC 1 0 0 -1 0 0;
ESTIMATE 'AUDITORY-VISUAL, 10 SEC' TC 0 1 0 0 -1 0;
ESTIMATE 'AUDITORY-VISUAL, 15 SEC' TC 0 0 1 0 0 -1;
RUN;
### General Linear Models Procedure

Dependent Variable: REACTM

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>5</td>
<td>0.02554894</td>
<td>0.00510979</td>
<td>17.66</td>
<td>0.0001</td>
</tr>
<tr>
<td>Error</td>
<td>12</td>
<td>0.00347200</td>
<td>0.00028933</td>
<td></td>
<td></td>
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<tr>
<td>Corrected Total</td>
<td>17</td>
<td>0.02902094</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R-Square</th>
<th>C.V.</th>
<th>Root MSE</th>
<th>REACTM Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.880362</td>
<td>7.458622</td>
<td>0.01700980</td>
<td>0.22805556</td>
</tr>
</tbody>
</table>

Source          | DF | Type I SS | Mean Square | F Value | Pr > F |
<table>
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<tr>
<td>TC</td>
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<td>0.0001</td>
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<td>0.0001</td>
</tr>
</tbody>
</table>

| Parameter       | Estimate | T for H0: Parameter=0 | Pr > |T| | Std Error of Estimate |
|-----------------|----------|-----------------------|-------|-------|-----------------------|
| AUDITORY-VISUAL | -0.07233333 | -9.02 | 0.0001 | 0.00801850 |
| AUDITORY-VISUAL, 5 S | -0.08333333 | -6.00 | 0.0001 | 0.01388844 |
| AUDITORY-VISUAL, 10 | -0.08066667 | -5.81 | 0.0001 | 0.01388844 |
| AUDITORY-VISUAL, 15 | -0.05300000 | -3.82 | 0.0025 | 0.01388844 |