1. Give two examples of each of the three types of spatial data: geospatial data, lattice data and spatial point patterns. You examples could be those in the literature (books and papers) but cannot be those given in the class or those in the handouts. If your examples are from the literature, please provide the complete citation of the sources where you obtained data. You can also construct your own examples.

2. Suppose we observe a process $Y(s)$ for s in the interval [0, 1]. Suppose this process has a constant unknown mean and an exponential covariogram

$$C(Y(s), Y(s+h)) = 2\exp(-5|h|).$$

We observe $Y(0) = 1.5$, $Y(1) = 0.5$, and predict $Y(s)$ for $s = 0.02, 0.04, \cdots, 0.98$ using the BLUP or the ordinary kriding predictor.

- Plot the ordinary kriging prediction $\hat{Y}(s)$ versus s.
- Plot the ordinary kriging variance $\sigma^2(s)$ versus s.

You can use some existing R packages or the one of the following methods. One method is to use the following formula:

$$\boldsymbol{\lambda} = V^{-1}(\boldsymbol{k} + m\mathbf{1}) \tag{1}$$
$$m = (1 - \mathbf{1}'V^{-1}\boldsymbol{k})/(\mathbf{1}'V^{-1}\mathbf{1}). \tag{2}$$

$$\hat{Y}(s) = \lambda Y(s_1) + (1-\lambda)Y(s_2).$$

Another method is to following the following steps:

(a) For any $s \in [0, 1]$, find the λ that minimizes the MSE

$$E[(\hat{Y}(s) - Y(s))^2].$$

(Hint: Write

$$MSE = \lambda^2 E(Y(s_1)-Y(s_2))^2 + 2\lambda Cov(Y(s_1)-Y(s_2), Y(s_2)-Y(s)) + E(Y(s_2)-Y(s))^2.$$

Then express the covariance and mean squared difference in terms of the covariogram. For example, $E(Y(s_1) - Y(s_2))^2 = 4(1 - \exp(-5))$.

(b) For any s, express the prediction variance in terms of the covariance and the optimal λ. The prediction variance is the minimal MSE.

3. Suppose Y_1 and Y_2 represent the temperatures at two time points, t_1 and t_2, respectively, and have a bivariate normal distribution with means $EY_i = \mu_i$, variances $Var(Y_i) = \sigma_i^2$, $i = 1, 2$, and correlation coefficient $0 < \rho < 1$.

(a) If the observed temperature at time point t_1 is T, show that the simple kriging prediction of Y_2 is

$$\hat{Y}_2 = \mu_2 + \frac{\sigma_2 \rho}{\sigma_1}(T - \mu_1).$$

(b) Show that if the temperature at time t_1 is above the mean, i.e., $P(Y_1 < T) > 0.5$, show that the predicted value given by the simple kriging is always above its mean but less extreme than T in the sense that

$$1/2 < P(Y_2 < \hat{Y}_2) < P(Y_1 < T).$$

For example, if T is the 95th percentile of Y_1, i.e., $P(Y_1 < T) = 0.95$, then, $1/2 < P(Y_2 < \hat{Y}_2) < 0.95$ which means $\hat{Y}_2$ is lower than the 95th.

(c) State what happens if the the observed temperature is below the mean.

The phenomena discussed above is call regression.