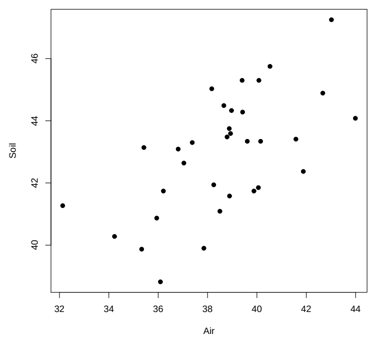

Figure 1: Scatter plot of soil temperature against air temperature

The purpose of this homework assignment is to enhance your understanding of cokriging and to learn to use the R function *c*ok to carry out cokriging. The primary spatial variable is the soil temperature and the auxiliary variable is the air temperature. Sample data revealed that the two variables are correlated, as shown in Figure 1. Let us apply the following model

$$Y_1(\boldsymbol{s}) = a + bY_2(\boldsymbol{s}) + e(\boldsymbol{s}), \tag{1}$$

where $Y_1(\boldsymbol{s})$ and $Y_2(\boldsymbol{s})$ represent the soil and temperature at location $\boldsymbol{s}$, respectively, $e(\boldsymbol{s})$ is a stationary process and independent of the process $Y_2(\boldsymbol{s})$. The predictor for $Y_1(\boldsymbol{s}_0)$ is

$$\hat{Y}_1(\boldsymbol{s}_0) = a + bY_2(\boldsymbol{s}_0) + \hat{e}(\boldsymbol{s}_0), \tag{2}$$

where $\hat{e}(\boldsymbol{s}_0)$ is the ordinary kriging predictor for $e(\boldsymbol{s}_0)$ based on the residuals $e(\boldsymbol{s}_i) = Y_1(\boldsymbol{s}_i) - a - bY_2(\boldsymbol{s}_i)$.

1. Let $C_{11}(h)$ and $C_{22}(h)$ denote the covariogram of $Y_1(\boldsymbol{s})$ and $Y_2(\boldsymbol{s})$, respectively, and $C(\boldsymbol{s})$ denote the covariogram of the error $e(\boldsymbol{s})$. Also let $C_{12}(h)$ denote the cross-covariogram of $Y_1(\boldsymbol{s})$ and $Y_2(\boldsymbol{s})$. Show that

$$C_{11}(h) = b^2C_{22}(h) + C(h), \quad C_{12}(h) = bC_{22}(h). \tag{3}$$

2. Use $a = 1.3389, b = 0.8458$ to calculate the residuals $e(\boldsymbol{s}_i) = Y_1(\boldsymbol{s}_i) - a - bY_2(\boldsymbol{s}_i)$. Fit an exponential covariogram model to the residuals by the maximum likelihood method.

3. Fit an exponential covariogram model to the air temperature $Y_2(\boldsymbol{s})$ through the maximum likelihood method. You then have an estimated $C_{22}(h)$.

4. We now use model (2) to find the cokriging prediction for the soil temperature at the last or the 39th location using the air temperature of all 39 locations and the soil temperature at the first 38 locations.

5. As shown in class, the cokriging prediction should equal the the following prediction

$$\hat{y}_{39} = 1.3389 + 0.8458Y_{air,39} + \hat{e}_{39},$$

where $\hat{e}_{39}$ is the ordinary kriging for $e(\boldsymbol{s}_{39})$ based on the residuals at the first 38 locations. You should find that the two predictions are equal.