

1. Give two examples of each of the three types of spatial data: geospatial data, lattice data and spatial point patterns. Your examples could be those in the literature (books and papers) but cannot be those given in the class or those in the handouts. If your examples are from the literature, please provide the citation of the literature. You can also construct your own examples.
2. Suppose we observe a process $Y(s)$ for s in the interval $[0, 1]$. Suppose this process has a constant unknown mean and an exponential covariogram

$$C(Y(s), Y(s+h)) = 2 \exp(-5|h|).$$

We observe the process at 2 locations, $s_1 = 0$, and $s_2 = 1$, and want to predict $Y(s)$ for s between $[0, 1]$ using a BLUP

$$\hat{Y}(s) = \lambda Y(s_1) + (1 - \lambda)Y(s_2).$$

- (a) For any $s \in [0, 1]$, find the λ that minimizes the MSE

$$E[(\hat{Y}(s) - Y(s))^2].$$

(Hint: Write

$$MSE = \lambda^2 E(Y(s_1) - Y(s_2))^2 + 2\lambda \text{Cov}(Y(s_1) - Y(s_2), Y(s_2) - Y(s)) + E(Y(s_2) - Y(s))^2.$$

Then express the covariance and mean squared difference in terms of the covariogram. For example, $E(Y(s_1) - Y(s_2))^2 = 4(1 - \exp(-5))$.

- (b) For any s , express the prediction variance in terms of the covariance and the optimal λ . The prediction variance is the minimal MSE.
- (c) Plot the optimal λ and the optimal MSE against s for $s \in [0, 1]$.