Week 13

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34 Lecture 34

34.1 Normal Distributions (8.5)

- Situation: On many occasions data arises from repeated measurements of the same random variable that can be characterized by their mean and variance.

- Definition: A continuous random variable X is called a normal random variable if it has a probability density function of the form,

  We say that X has a normal distribution with parameters μ and σ^2 or that X is normally distributed with parameters μ and σ^2, denoted by $X \sim \mathcal{N}(\mu, \sigma^2)$.

- Properties:

  1. $E(X) =$ $\qquad\qquad Var(X) =$

  2. PDF has

  3. $median(X) =$

- Standard Normal RV: A normal random variable with $\mu = 0$ and $\sigma^2 = 1$ is called a standard normal random variable and is said to have the standard normal distribution. We denote the PDF of a standard normal random variable as ϕ and CDF as Φ. Thus,

- Standardizing a Normal RV: Let X be a normally distributed RV with parameters μ and σ^2. Then,

- Obtaining Normal Probabilities from Φ: Let X be a normally distributed RV with parameters μ and σ^2. Then,

- Examples:

  1. Let $Z \sim \mathcal{N}(0,1)$. Find the following

     (a) $P(Z \leq 1.92) =$

     (b) $P(Z < -2.61) =$

(c) $P(-0.34 < Z < 0.91) =$

(d) $P(Z = -0.55) =$

2. Let $Z \sim \mathcal{N}(0,1)$. Find z_0.

   (a) $P(Z \leq z_0) = 0.97$

   (b) $P(Z > z_0) = 0.33$

   (c) $P(-z_0 < Z < z_0) = 0.77$

3. Suppose the height of an adult man is normally distributed with mean 69 inches and standard deviation 2.5 inches.

   (a) What percent of men are at least 72 inches tall?

   (b) What percent of men are between 60 inches and 72 inches tall?

   (c) How tall must a man be to be in the tallest 10% of all adult men?

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35.1 Normal Distributions Continued (8.5)

- 68-95-99.7 Rule:

  1. $P(|X - \mu| \leq \sigma)$ = probability within 1 standard deviation of the mean =

  2. $P(|X - \mu| \leq 2\sigma)$ = probability within 2 standard deviation of the mean =

  3. $P(|X - \mu| \leq 3\sigma)$ = probability within 3 standard deviation of the mean =

- Sum of Independent Normal RVs: Let $X_1, \ldots, X_m$ be independent RVs with $X_j \sim \mathcal{N}(\mu_j, \sigma_j^2)$ for $1 \leq j \leq m$, and let $a, b_1, \ldots, b_m$ be real numbers. Then,

  In particular,

  and

- Examples:

1. $X_1 \sim \mathcal{N}(\mu = 2, \sigma^2 = 9)$, $X_2 \sim \mathcal{N}(\mu = 3, \sigma^2 = 4)$, and X_1 and X_2 are independent.
   (a) Find the probability that $5X_1 + 2$ is greater than 24.

   (b) Find the probability that $5X_1 - 2X_2$ is greater than 15.

   (c) Find the probability that $X_1 < 0$ or $X_2 < 0$.

2. The annual rate of return for a share of a specific stock has a normal distribution with mean 10% and standard deviation 12%. If you buy 100 shares of the stock at a price of \$60 per share, what is the probability that after a year, your net profit from that investment is at least \$750? (Ignore transaction costs and assume that there is no annual dividend.)

3. $X \sim \mathcal{N}(\mu = 10, \sigma^2 = 9)$
   (a) With probability 68%, X lies in

   (b) With probability 95%, X lies in

   (c) With probability 99.7%, X lies in