Week 11

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28 Lecture 28

28.1 Probability Density Functions (8.1, 8.3)

- Continuous random variable: A random variable X is called a continuous random variable if

- Definition of PDF: Let X be a continous random variable. f_X is called a probability density function (PDF) of X if, for all real numbers $a < b$,

- Properties of PDF: Let X be a continuous random variable and f_X its PDF. Then,
  1. $f_X(x) \geq$
  2. $\int_{-\infty}^{\infty} f_X(x)dx =$

- Fundamental Probability Formula for Univariate Continuous Random Variables: Suppose that X is a continuous random variable having a PDF. Then, for any subset A of real numbers,

  In particular,

28.2 Cumulative Density Functions (8.2)

- Definition of CDF: Let X be a random variable. Then the cumulative distribution function (CDF) of X, denoted F_X, is the real-valued function defined on $\mathcal{R}$ by,

- Properties of CDF: Let X be a random variable either discrete or continuous and F_X its CDF. Then,

  1. F_X is

  2. F_X is

  3. $F_X(-\infty) = \lim_{x \to -\infty} F_X(x) =$

  4. $F_X(\infty) = \lim_{x \to \infty} F_X(x) =$

- Computing Probabilities with CDF: Let X be a random variable. Then, for all real numbers $a < b$,

  1. $P(a < X \leq b) =$

  2. $P(a \leq X \leq b) =$

  3. $P(a < X < b) =$

  4. $P(a \leq X < b) =$

28.3 Relationship and Examples (8.3)

- PDF is the derivative of CDF: Let X be a continuous random variable. If F'_X exists and is continuous except possibly at a finite number of points, then X has a PDF given as,

- Examples:

  1. Let X be a continuous random variable with PDF

     (a) Draw a labeled sketch of PDF $f_X(z)$.

     (b) Find the value of the constant c.

     (c) Find the cumulative distribution function $F_X(x)$.

     (d) Draw a labeled sketch of the CDF $F_X(x)$.

     (e) Compute $P(2 < X \leq 4)$.

2. Which of the following could be a PDF?

   (a) $f_{X1} =$

   (b) $f_{X2} =$

3. Let X be the battery's lifetime in years. The CDF of X is

   (a) Find the density function of X.

   (b) Find the median of X.

   (c) What is the probabiilty that the battery will last more than a year?

   (d) Given that the battery has lasted half a year, what is the probability that it will last another half a year?

29 Lecture 29

29.1 Expectation of Continuous Random Variables (10.1, 10.2)

- Definition: The expected value of a continuous random variable X, denoted $E(X)$, is defined by

- Properties of expectations: Suppose a, b, c are constants, and X, Y are continuous random variables.

  1. $E(c) =$
  2. $E(cX) =$
  3. if X has a finite expectation, $E(a + bX) =$
  4. if X and Y has finite expectations, $E(X + Y) =$
  5. if X and Y are independent, $E(XY) =$

- Fundamental Expected-Value Formula (Continuous Case): Let X_1, ..., X_m be continuous random variables with a joint PDF and leg g be a real-valued function of X_1, ..., X_m. Then $g(X_1, ..., X_m)$ has expectation,

  In particular,

29.2 Variance and Median of Continuous RVs (10.3)

- Definition of Variance: The variance of a continuous random variable X, denoted $Var(X)$, is defined as,

- Definition of Median: The median of a continuous random variable X, denoted $median(X)$, is defined as,

29.3 Examples

1. Let X be a continuous random variable with PDF given as,

   (a) Find $E(X)$ and $Var(X)$.

   (b) Find $E(1/X)$.

   (c) Let $Y = 3X + 10$, find $E(Y)$ and $Var(Y)$.