Week 7

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18 Lecture 18

18.1 Hypergeometric Random Variable (5.4)

- Definition: Consider a population of total size N, in which a proportion p of the population have a specified attribute (and $1-p$ does not). Suppose that a random sample of size n (where $n \leq N$) is taken without replacement from the population. Let X denote the number of members sampled that have the specified attribute. Then the PMF of X is

  X is called a hypergeometric random variable and is said to have the hypergeometric distribution with parameters N, n, and p, denoted as $X \sim \mathcal{H}(N, n, p)$

- Alternative Definition: Consider a population of total size N, in which M members of the population have a specified attribute (and $N - M$ does not). Suppose that a random sample of size n (where $n \leq N$) is taken without replacement from the population. Let X denote the number of members sampled that have the specified attribute. Then the PMF of X is

- Mean and Variance: Let $X \sim \mathcal{H}(N, n, p)$

  1. $E(X) =$

2. $Var(X) =$

- Binomial approximation to Hypergeometric for $N >> n$: Suppose $X \sim \mathcal{H}(N, n, p)$. Then,

  As a rule of thumb, we may adequately approximate the hypergeometric distribution with a binomial distribution if $N \geq 10n$ or $0.1N \geq n$, that is the sample size does not exceed 10% of the population.

- Examples

  1. There are 10 quarters and 4 nickels. You choose 5 coins at random and without replacement. Let Y=# nickels selected.
     (a) What is the probability that at least 2 nickels are selected?

     (b) What is the expected number of nickels which are selected? Standard deviation?

  2. If there are 10,000 quarters and 4,000 nickels. You choose 5 coins at random and without replacement. What is the probability that at least 2 nickels are selected?

19 Lecture 19

19.1 Geometric Random Variable (5.6)

- Geometric situation: A geometric situation is when we are waiting for the first success in repeated Bernoulli trials.
- Definition: Consider a geometric situation in which repeated Bernoulli trials are performed with probability of success p. Let X denote the number of trials up to and including the first success. Then the PMF of the random variable X is

  The random variable X is called a geometric random variable and is said to have the geometric distribution with parameter p, denoted as $X \sim \mathcal{G}(p)$.
- Mean and variance: Let $X \sim \mathcal{G}(p)$.
  1. $E(X) =$
  2. $Var(X) =$
- Tail probabilities of a Geometric Random Variable: Suppose that X has the geometric distribution with parameter p. Then
- lack-of-memory (memoryless) property:
  - A discrete random variable X is said to possess the lack-of-memory property if
  - A positive-integer valued random variable has the lack-of-memory property if and only if it has a geometric distribution.

- Example

1. Suppose that 20% of a group of people have hazel eyes.

   (a) What is the probability that the 5th passenger boarding a plane is the first one having hazel eyes?

   (b) What is the average number of passengers you have to observe until you see the first one with hazel eyes? Standard deviation?

   (c) What is the probability that you have to observe at least 5 passengers in order to see the first one with hazel eyes?

   (d) If the first 5 passengers you observed don't have hazel eyes, what is the probability that you have to observe at least 3 more passengers to find the first one with hazel eyes?

19.2 Negative Binomial Random Variable (5.6)

- Negative Binomial situation: A negative binomial situation is when we are waiting for the rth success.

- Definition: Consider a negative binomial situation in which repeated Bernoulli trials are performed with success probability p. Let X denote the number of trials up to and including the rth success. Then the PMF of the random variable X is

  The random variable X is called a negative binomial random variable and is said to have the negative binomial distribution with parameters r and p, denoted $X \sim \mathcal{NB}(r, p)$.

- Mean and variance: Let $X \sim \mathcal{NB}(r, p)$.

  1. $E(X) =$

  2. $Var(X) =$

- Example

  1. A vending machine contains cans of grapefruit juice that cost 75 cents each, but it is not working properly. The probability that it accepts a coin is 0.1. You have a quarter and 5 dimes.

     (a) What is the probability that you should try the coins exactly 50 times to get a can of grapefruit juice?

     (b) On average, how many times should she try the coins to get a can of grapefruit juice? Standard deviation?