

Week 4

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9 Lecture 9

9.1 Law of Total Probability (4.2)

- Partition of a sample space: Events A_1, A_2, \dots form a partition of a sample space Ω if they are
 - a) mutually exclusive: $A_m \cap A_n = \emptyset$ for all $m \neq n$.

and b) exhaustive: $\bigcup_n A_n = \Omega$.

- **Law of Total Probability:** Suppose that A_1, A_2, \dots form a partition of the sample space. Then, for each event B ,

In particular, if E is an event, then

- Example: If 5% of the men and 0.25% of women are color blind, what is the probability that a randomly selected person is color blind if we are equally likely to select either a man or woman?

9.2 Bayes's Rule (4.4)

- **Bayes's Rule:** Suppose that A_1, A_2, \dots form a partition of the sample space. Then, for each event B ,

where A_j can be any one of the events A_1, A_2, \dots

In particular, if E is an event, then

- **Example:** If 7.0% of a population has lung disease. Of those with lung disease, 90.0% are smokers; and of those without lung disease, 74.7% are non-smokers. What is the probability that a randomly selected smoker has lung disease?

10 Lecture 10

10.1 Independent Events (4.3)

- **Independence for two events:** Let A and B be events of a sample space with $P(A) > 0$. B is independent of A if either of the following two equivalent definitions (I and II) are satisfied.

I) $P(B|A) = P(B)$.

or II) $P(A \cap B) = P(A)P(B)$. (Special Multiplication Rule)

- **Independence for three or more events:** Events A_1, A_2, \dots, A_N are mutually independent (or simply independent) if it satisfies the general multiplication rule,

- **Pairwise independent events:** Events A_1, A_2, \dots, A_N are pairwise independent if

- Distinction of Mutually exclusive vs. Independent Events:

- Examples:

1. Is anything independent with everything? (That is, is an event A of the sample space Ω independent with Ω itself?):
2. If A is independent with B , is A independent with B^c ? A^c independent with B ? A^c independent with B^c ?
3. Given an urn containing 5 red and 7 blue balls. Two balls are drawn at random. Is the event $A = \{1\text{st ball is red}\}$ independent with $B = \{2\text{nd ball is red}\}$?
 - i) if sampled with replacement?
 - ii) without replacement?
4. Suppose that two dice are rolled, one black and one grey. Let $A = \{\text{event of black die being even}\}$, $B = \{\text{event of gray die being even}\}$, and $C = \{\text{event the sum of the dice is even}\}$. Are the events A , B , and C pairwise independent or (mutually) independent?