Week 3

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7 Lecture 7

7.1 Counting Examples

- If 5 people wearing black suits and 5 wearing blue suits sit in a row in random order, what is the probability that no two person wearing the same color of suit sit together?

- During a poker game, you are dealt a 5-card hand at random. What is the probability you get a flush?

- What is the probability you get two pairs?

- How many ways can you order 3 digits randomly chosen from $0, 1, ..., 9$?

7.2 Conditional Probability (4.1)

- **Conditional Probability**: Let A and B be events of a sample space. The probability that event B occurs, given that event A occurs, is called a conditional probability. It is denoted $P(B|A)$, read as "probability of B given A".

- Examples: Consider the experiment of picking cards starting from a 52-card deck.

  1. What is the Probability of getting a 2 of Diamond given that you pick from the full deck of 52 cards?:

  2. What is the probability of getting a 2 of Diamond given that you pick from a deck of 0 cards?:

  3. What is the probability of getting a Queen of Diamond given that you pick from a deck of Face-only cards?:

  4. What is the probability of getting a 2 of Diamond given that you pick from a deck of Face-only cards?:

- **Conditional Probability Rule**: The conditional probability that event B occurs given that event A occurs is

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8.1 Probabilities of Conditional Probability

If $P(A) > 0$, then

1. $P(\emptyset|A) = 0$

2. $P(B|A) = P(B)$ if $A = \Omega$

3. Domination principle: if $B \subset C$, then $P(B|A) \leq P(C|A)$.

4. Complementation rule: $P(B^c|A) = 1 - P(B|A)$.

5. Law of Partitions: $P(B|A) = P(E \cap B|A) + P(E^c \cap B|A)$.

6. General Addition Rule: $P(B \cup C|A) = P(B|A) + P(C|A) - P(B \cap C|A)$.

7. Disjoint Addition: If $B \cap C = \emptyset$ then $P(B \cup C|A) = P(B|A) + P(C|A)$.

8. Inclusion-Exclusion principle: $P(B \cup C \cup E|A) = P(B|A) + P(C|A) + P(E|A) - P(B \cap C|A) - P(B \cap E|A) - P(C \cap E|A) + P(B \cap C \cap E|A)$.

8.2 Multiplication Rule (4.1, 4.2)

- **General Multiplication Rule**:

  i. Let A and B be events with $P(A) > 0$. Then

  ii. More generally, if $A_1, A_2, \ldots, A_N$ are events with $P(A_1 \cap A_2 \cap \cdots \cap A_{N-1}) > 0$, then

- Examples

  1. Polya's Urn: Suppose an urn contains seven black balls and five white balls. We draw two balls from the urn without replacement. Assuming that each ball in the urn is equally likely to be drawn, what is the probability that both balls drawn are black?

  2. If we draw three balls from the urn without replacement, what is the probability that all three balls drawn are black?

  3. Suppose that each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. What is the probability that none of the three men selects his own hat?