Week 12

John Daye

Stat 225, Fall 2005

30 Lecture 30

30.1 Uniform Distribution (8.4)

- Situation: The uniform distribution is used to model the situation when every value in a specified interval is equally likely. (It is the continuous analogue of equally likely elementary events.)

  1. X is a number chosen randomly from $[0, 1]$.

  2. A cellphone rings during the lecture, X is the exact time at which it rings.

- Definition: A continuous RV X is called a uniform random variable if for some finite interval (a, b), its PDF is constant on that interval and 0 elsewhere or, its value is equally likely to lie anywhere in that interval. We say that X has the uniform distribution on the interval (a, b) or that X is uniformly distributed on the interval (a, b), denoted as $X \sim \mathcal{U}(a, b)$.

- PDF and CDF: For $X \sim \mathcal{U}(a, b)$, it's PDF and CDF is given by,

- Expectation and Variance:

  1. $E(X) =$

  2. $Var(X) =$

- Example: Suppose you arrive at a bus stop at 10:00am. The bus will arrive at a time T uniformly distributed between 10:00am and 10:30am.

  1. What is the probability that you'll have to wait more than 10 minutes for the bus?

  2. If at 10:15am the bus has not yet arrived, what is the probability that you have to wait at least 10 more minutes?

  3. What is the probability that the bus will arrive exactly at 10:15am?

31 Lecture 31

31.1 Exponential (8.4)

- Situation: The exponential distribution is used to model waiting times in the continuous case. (It is the continuous analogue of geometric random variable.)

  1. X is the time until the next customer arrives at a bank.

  2. X is the time until a lightbulb burns out.

  3. X is the mileage you get out of one tank of gas.

- Definition: A continuous random variable X is called an exponential random variable if it has PDF of the form,

  We say that X has an exponential distribution with parameter λ or that X is exponentially distributed with parameter λ, denoted as $\mathcal{E}(\lambda)$.

- CDF: For $X \sim \mathcal{E}(\lambda)$, it's CDF is given by,

- Expectation and Variance:

  1. $E(X) =$

  2. $Var(X) =$

- Survival function: If the exponential random variable X is used to model a lifetime, then the probability of survival until time t is,

  This function is called the survival function for X.

- Lack-of-memory property: A positive continuous random variable has the lack-of-memory property if and only if it has an exponential distribution.

- Example: Suppose the time X that it takes a customer representative to help a customer at a phone hot line has an exponential distribution with mean 2.5 minutes.

  1. What is the probability that it takes the customer representative more than 3 minutes to help the next customer?

  2. What is the probability that it takes the customer representative less than 3 minutes to help each of the next 5 customers?

32 Lecture 32

32.1 Poisson Process (12.1)

- Counting process: Let $N(t)$ be the number of events that occurs by time t. Then $N(t)$ is a counting process.

- Definition: A counting process $\{N(t) : t \geq 0\}$ is said to be a Poisson Process with rate λ if

- Properties: Let $N(t)$ be a Poisson process with rate λ. Then,

  1. $N(0) =$

  2. $\{N(t) : t \geq 0\}$ has

  3. $N(t) - N(s) \sim$

  4. The waiting time between consecutive events has

  5. The waiting time between different events are

  6. Given the number of events that occur in a certain time interval, the exact arrival times of those events are

- Example: Suppose phone calls arrive at a calling center with an average rate of 3 per minute according to a Poisson process.

  1. Let X be the number of calls that come in between 10:05am and 10:09am. Find the distribution of X.

  2. Let Y be the waiting time between the second and third call. Find the distribution of Y.

  3. What is the probability that exactly four calls will arrive within the next 2 minutes?

  4. What is the probability that the next call arrives within 20 seconds and the second call at least 60 seconds after the first?

  5. Given that exactly 5 calls arrived in the last 2 minutes, what is the probability that they all arrived in the last minute?