

## Quiz 3

Stat 225, Fall 2005

Due Wednesday 10/05

Do 3 of the following problems.

1. You flip 3 fair coins. At least two are alike, and there is an even chance that the third is a head or tail. Therefore,  $P(\text{all alike}) = \frac{1}{2}$ . Is this true? Why?
2. Your company has been sued for accounting fraud. One of your junior accountant, Bob, has given unfavorable testimony that could result in a guilty sentence. The prosecuting lawyer argues that since the probability that Bob gives a truthful testimony, given that your company is guilty, is 0.9, your company must be guilty, with the same probability of 0.9, given Bob's testimony.
  - (a) Describe the situation symbolically, using T to stand for Bob's testimony being truthful and G your company is guilty.
  - (b) Argue why the prosecuting lawyer might be incorrect.
  - (c) When would the argument be correct?
3. Most financial pricing methods are based upon the Arbitrage-free principle, that essentially states that there cannot be any pricing discrepancies in an efficient market to ensure a probabilistic sure-win for any individual. Thus, a rule-of-thumb is to price an instrument (merchandise) based upon its expected value.

In a lottery every week, 2,000,000 tickets are sold for \$1 apiece. If 4000 of these tickets pay off \$30 each, 500 pay off \$800 each, one ticket pays off \$1,200,000, and no ticket pays off more than one prize,

  - (a) What is the expected value of the winning amount for a player with a single ticket?
  - (b) Based upon part (a), what is your expected profit if you purchased the ticket for only \$0.50?
  - (c) Based upon the Arbitrage-free principle, is \$0.50 a reasonable price if the lottery ticket market is efficient?

4. Standard deviation is a widely used measure of risk.

You are about to purchase a stock, hold it, and sell it on a fixed future date. There are two stocks available, that is, A and B. Both start at \$10 per share on the purchase date, and each stock gains/loses an amount at the selling date according to a certain random variable. Stock A is described by the random variable  $X$  with probability mass function,

$$p(x) = \begin{cases} 1/2 & \text{if } x = 2 \\ 1/2 & \text{if } x = 0 \end{cases}$$

That is, stock A may increase by \$2 per share or stay the same on the selling date, with an equal probability of 1/2. Stock B is described by  $Y$  with the probability mass function,

$$p(y) = \begin{cases} (|y + 3| + 1)/28 & \text{if } y = -3, -2, -1, 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

- (a) What is the expected price change of each stock on the selling date?
- (b) What is the standard deviation of the price change for each stock?
- (c) Using standard deviation as your measure of risk, determine which stock is better to purchase if you are risk-averse? Why?