Assignment 1 (Due next Friday (09/07/00))

1. Given $A = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$.

(a) Verify that A is a positive definite matrix.

(b) Find the eigenvalues and eigenvectors of A.

(c) Derive and verify the spectral decomposition of A.

(d) Find $\max_{\vec{x} \neq 0} \frac{\vec{x}' A \vec{x}}{\vec{x}' \vec{x}}$ and $\min_{\vec{x} \neq 0} \frac{\vec{x}' A \vec{x}}{\vec{x}' \vec{x}}$.

(Hint: Page 105)

2. Suppose a discrete random vector $\vec{X}' = (X_1, X_2)$ has the following distribution

x_1	x_2	$p(x_1, x_2)$
0	0	0.1
0	1	0.2
1	0	0.3
1	1	0.4

Find the population mean, the variance-covariance matrix and the correlation coefficient matrix of $\vec{X}$.

(Hint: Page 72)

3. Suppose $\vec{X}' = (X_1, X_2)$ follows the bivariate normal distribution $N_2(\vec{\mu}, \Sigma)$ where $\vec{\mu}' = (1, 1)$ and $\Sigma = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$.

(a) Write out the density function of $\vec{X}$ explicitly.

(b) Let $\vec{Y} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$. Find the distribution of $\vec{Y}$.

(c) Find the conditional distribution of X_1, given that $X_2 = x_2$.

(d) Determine and sketch the constant-density contour that contains 50% of the probability(Specify the center and the axies)

4. You are suggested to read through the splus tutorial material written by Venables and Smith and practice the commands and examples there.