Midterm 1 Solution

1.
a. 3.3375
b. 1.950778
c.
2.

$$\frac{\binom{6}{2}\binom{4}{2}}{\binom{10}{4}} = 3/7 = 0.4286$$

3.
a. (.40)(1-.95)=.02
b. (.40)(.95) + (.40)(.97) + (.20)(.90)=.948
c. .02/(1-.948)=0.02/0.052=.3846
4.
a. Let X_{2A} be the number of breakdowns of A in two weeks. X_{2A} has a Poisson distribution with $\lambda = 2 \times 1 = 2$.

$$P(X_{2A} = 0) = .135 \text{ Table A.2}$$

b. Let X_B be the number of breakdowns of B in one week. X_B follows a Poisson distribution with $\lambda = 2$.

$$P(X_B \geq 4) = 1 - P(X \leq 3) = 1 - F(3;2) = 1 - .857 = .143$$

c. Let X_A be the number of breakdowns of A in one week. $X_A \sim \text{Poisson}(1)$.

$$P(X_A{+}X_B = 3) = P(X_A = 0, X_B = 3){+}P(X_A = 1, X_B = 2){+}P(X_A = 2, X_B = 1){+}P(X_A = 3, X_B = 0)$$

$$= P(X_A = 0)P(X_B = 3){+}P(X_A = 1)P(X_B = 2){+}P(X_A = 2)P(X_B = 1){+}P(X_A = 3)P(X_B = 0)$$

=.224
Another approach:
Let X_{A+B} be the total number of breakdowns in the office in a week. Claim that X_{A+B} follows Poisson distribution with paramer $\lambda = 1 + 2 = 3$ (why?).

$$P(X_{A+B} = 3) = F(3;3) - F(2;3) = .224$$

d. Since A and B are independent, $E(X_A)$=1.
5.

a. $\int_{-1}^{1} f(x)dx = \int_{-1}^{0} 1/4dx + \int_0^1 (1/4 + kx)dx = 1$, $k = 1$.
b.

$$E(X) = \int_{-1}^{0} x/4dx + \int_{0}^{1} (1/4 + x)xdx = 1/3$$

$$E(X^2) = \int_{-1}^{0} x^2/4dx + \int_{0}^{1} x^2(1/4 + x)dx = 5/12$$

mean is 1/3, and variance is $E(X^2) - (E(X))^2 = 11/36$
C.

$$P(X \geq 1/2 \mid X > 0) = \frac{P(X \geq 1/2)}{P(X > 0)}$$

$$P(X \geq 1/2) = \int_{1/2}^{1} (1/4 + x)dx = 1/2$$

$$P(X > 0) = 1 - P(X \leq 0) = 1 - 1/4 = 3/4$$

Hencen $P(X \geq 1/2 \mid X > 0) = \frac{1/2}{3/4} = 2/3$.
6.
$X \sim N(6, 0.06)$
a.

$$P(5.902 < X < 6.098) = P(\frac{5.902 - 6}{0.06} < Z < \frac{6.098 - 6}{0.06})$$
$$= P(-1.633 < Z < 1.6333) = 0.9484 - 0.0516 = 0.8968$$

b. Let Y be the number of rods that are not acceptible. Y is a binomial random variable with $n = 25$ and $p = 1 - 0.8968 \approx= 0.10$

$$P(Y \geq 3) = 1 - B(2; 25, 0.10) = 1 - .537 = .463$$

c. Now we need to calculate new σ.

$$P(5.902 < X < 6.098) = P(\frac{5.902 - 6}{\sigma} < Z < \frac{6.908 - 6}{\sigma}) = 0.99$$

Hence $\frac{-.098}{\sigma}$ is the .5th percentile, that is -2.575.

$$\frac{-.098}{\sigma} = -2.575$$

So, σ=0.098/2.575=0.038.