

3.82

In proof testing of circuit boards, the probability that any particular diode will fail is 0.01. Suppose a circuit board contains 200 diodes.

- a. How many diodes would you expect to fail, and what is the standard deviation of the number that are expected to fail?
- b. What is the (approximate) probability that at least four diodes will fail on a randomly selected board?
- c. If five boards are shipped to a particular customer, how likely is it that at least four of them will work properly? (A board works properly only if all its diodes work).

Let X be the number of diodes on a board that fail.

a. $E(X) = np = (200)(0.01) = 2,$

$$V(X) = np(1 - p) = (200)(0.01)(0.99) = 1.98,$$

$$\sigma_X = 1.407$$

b. X has approximately a Poisson distribution with $\lambda = 2$, so

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - F(3; 2) = 1 - .857 = 0.143$$

**c. $P(\text{board works properly}) = P(\text{all diodes work})$
 $= P(X = 0) = F(0; 2) = .135.$**

Let Y be the number among the five boards that work, a binomial r.v. with $n = 5$. Then

$$P(Y \geq 4) = P(Y = 4) + P(Y = 5)$$

$$\begin{aligned} &= \binom{5}{4} (1 - 0.135)^1 (0.135)^4 + \binom{5}{5} (1 - 0.135)^0 (0.135)^5 \\ &= 0.00144 + 0.00004 = 0.00148 \end{aligned}$$