4.3 Suppose the error involved in making a certain measurement is a continuous r.v. with pdf
\[ f(x) = \begin{cases} 
0.09375(4 - x^2) & -2 \leq x \leq 2 \\
0 & \text{otherwise}
\end{cases} \]

a. Sketch the graph of \( f(x) \).
b. Compute \( P(X > 0) \).
c. Compute \( P(-1 < X < 1) \).
d. Compute \( P(X < -.5 \text{ or } X > .5) \)

\[ P(X > 0) = \int_{0}^{2} .09375(4 - x^2)dx = .09375(4x - \frac{x^3}{3}) \bigg|_{0}^{2} = .5 \]
(\textbf{Is there a shortcut to get the answer?})

c. \[ P(-1 < X < 1) = \int_{-1}^{1} .09375(4 - x^2)dx = .6875 \]
d \[ P(X < -.5 \text{ or } X > .5) = 1 - P(-.5 \leq X \leq .5) = 1 - \int_{-.5}^{.5} .09735(4 - x^2)dx = 1 - .3672 = .6328 \]
4.4 Let $X$ denote the vibratory stress (psi) on a wind turbine blade at a particular wind speed in a wind tunnel. The article “Blade Fatigues Life Assessment with Application to VAWTS” proposes the Rayleigh distribution, with pdf

$$f(x; \theta) = \begin{cases} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

as a model of the distribution of $X$

a. Verify that $f(x; \theta)$ is a legitimate pdf

b. Suppose $\theta$ is 100, what is the probability that $X$ is at most 200? less than 200? At least 200?

c. What is the probability that $X$ is between 100 and 200?

d. Give the expression for $P(X \leq x)$

a.

$$\int_{-\infty}^{+\infty} f(x; \theta) = \int_{0}^{+\infty} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} \, dx = -e^{-x^2/2\theta^2} \bigg|_{0}^{+\infty} = 0 - (-1) = 1$$

b.

$$P(X \leq 200) = \int_{0}^{200} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} \, dx = -e^{-x^2/2\theta^2} \bigg|_{0}^{200} = -.1353 - (-1) = .8647$$

c.

$$P(100 \leq X \leq 200) = \int_{100}^{200} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} \, dx = -e^{-x^2/2\theta^2} \bigg|_{100}^{200} = .4712$$

d. For $x \leq 0$, $P(X \leq x) = 0$. For $x > 0$,

$$P(X \leq x) = \int_{0}^{x} \frac{y}{\theta^2} \cdot e^{-y^2/(2\theta^2)} \, dy = -e^{-y^2/2\theta^2} \bigg|_{0}^{x} = 1 - e^{-x^2/2\theta^2}$$