1. Take home exam: 4pm April 2 – 9am April 3.
2. Must be finished independently.

1(6pts). Suppose $Y_{n1} = X_{n \times p} \beta_{p \times 1} + \epsilon_{n \times 1}$ is a linear model with $\epsilon \sim N(0, \sigma^2 I_n)$. Let $X$ be partitioned into two parts column-wise, that is, $X = (X_1, X_2)$, where $X_1$ is a $n \times r$ matrix and $X_2$ is $n \times (p - r)$. Suppose $\text{rank}(X) = \text{rank}(X_1) = r$. Let $a$ be a $p$-dimensional vectors, and $a$ is also partitioned into $a_1$ and $a_2$ correspondingly, that is, $a' = (a_1', a_2')$. Show that $a' \beta$ is estimable if and only if $a_2 = X_2'X_1(X_1'X_1)^{-1}a_1$.

2(7pts). Suppose $\{Y_{11}, Y_{12}\}$, $\{Y_{21}, Y_{22}\}$, and $\{Y_{31}, Y_{32}\}$ are independent measurements of the angles of a triangle subject to error. The mean of measurement error is zero and the variance is $\sigma^2$.

a) Obtain the least squares estimates of the angles.
b) Test if the triangle is an equilateral triangle.

3(7pts). Let $Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \epsilon_i$, $i = 1, \ldots, n$, $(n > 3)$, where $\sum X_i = 0$, $\sum X_i^3 = 0$, and $\epsilon_i$’s are iid $N(0, \sigma^2)$. Assume $\beta_2 \neq 0$. We wish to find the value $x^*$, at which the mean response attains an extremum (a maximum or minimum).

a) Find an estimate of $x^*$.
b) Test if $x^* = c$, where $c$ is a given constant.