Stat553S08 Midterm 2 (Total 20 Points)

1. Take home exam: 4pm April 2 – 9am April 3.
2. Must be finished independently.

1(6pts). Suppose $Y_{n\times 1} = X_{n\times p}\beta_{p\times 1} + \epsilon_{n\times 1}$ is a linear model with $\epsilon \sim N(0, \sigma^2 I_n)$. Let X be partitioned into two parts column-wise, that is, $X = (X_1, X_2)$, where X_1 is a $n \times r$ matrix and X_2 is $n \times (p - r)$. Suppose rank(X)=rank(X_1)= r. Let a be a p-dimensional vectors, and a is also partitioned into a_1 and a_2 correspondingly, that is, $a' = (a_1', a_2')$. Show that $a'\beta$ is estimable if and only if $a_2 = X_2'X_1(X_1'X_1)^{-1}a_1$.

2(7pts). Suppose $\{Y_{11}, Y_{12}\}$, $\{Y_{21}, Y_{22}\}$, and $\{Y_{31}, Y_{32}\}$ are independent measurements of the angles of a triangle subject to error. The mean of measurement error is zero and the variance is σ^2.
a) Obtain the least squares estimates of the angles.
b) Test if the triangle is an equilateral triangle.

3(7pts). Let $Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \epsilon_i$, $i = 1, \ldots, n, (n > 3)$, where $\sum X_i = 0$, $\sum X_i^3 = 0$, and ϵ_i's are iid $N(0, \sigma^2)$. Assume $\beta_2 \neq 0$. We wish to find the value x^*, at which the mean response attains an extremum (a maximum or minimum).
a) Find an estimate of x^*.
b) Test if $x^* = c$, where c is a given constant.