Assignment 1

Due Next Thursday 5pm

1. Let A be an $n \times n$ nonsingular square matrix, x be a n dimensional vector and c be a constant.
(a) Show that

$$\det \begin{pmatrix} A & x \\ x' & c \end{pmatrix} = |\mathrm{A}|(c - x'A^{-1}x)$$

(b) Show that $|A + xx'| = |A|(1 + x'A^{-1}x)$.

2. Let A be an $m \times n$ matrix.
(a) Show that $\mathrm{rank}(AA') = \mathrm{rank}(A'A) = \mathrm{rank}(A) = \mathrm{rank}(A')$.
(b) Show that $\mathcal{R}(AA') = \mathcal{R}(A)$.

3. Find the eigenvalues and eigenvectors of the matrix given below by hand.

$$\begin{pmatrix} 4 & 2 & 0 & 4 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 4 & 0 & 7 \end{pmatrix}$$

4. Show that a symmetric matrix A is of rank 1 if and only if $A = aa'$ where a is a nonzero vector.

5. Let A be a positive definite matrix.
(a) Show that

$$\max_{x \neq 0} \frac{(b'x)^2}{x'Ax} = b'A^{-1}b$$

where b is a given vector.
(b) $(x'Ay)^2 \leq (x'Ax)(y'Ay)$ for any x and y.
(c) $(x'y)^2 \leq (x'Ax)(y'A^{-1}y)$ for any x and y.

6. Let $A = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix}$ be an $n \times n$ matrix and X be a $r \times r$ sub-matrix. Suppose rank(A)=$r < n$ and X is nonsingular.
(a) Show that $W = ZX^{-1}Y$.
(b) Show that $\begin{pmatrix} X^{-1} & 0 \\ 0 & 0 \end{pmatrix}$ is a generalized inverse of A.