1. KNNL Problem 2.17

Solution: Since the analyst concluded $H_a : \beta_1 \neq 0$, the $p$-value $= 0.033 < \alpha$ and the analyst rejected $H_0$. The significance level $\alpha$ was greater than 0.033, probably $\alpha = 0.05$. If the $\alpha$ level had been 0.01, I would fail to reject $H_0$.

2. KNNL Problem 2.22

Solution: Theoretically, it is possible that $R^2 = 0$ for part of the data; i.e. the scatterplot shows no pattern at all for the first ten cases, but $R^2 \neq 0$ for the complete dataset; i.e. the scatterplot shows pattern for the complete set. It is also possible that $R^2 \neq 0$ for part of the data; i.e. the scatterplot shows some pattern for the first ten cases, yet the pattern disappears when you plot all the cases.

3. Given that $R^2 = SSM/SST$, it can be shown that $R^2/(1 - R^2) = SSM/SSE$. If you have $n = 22$ cases and $R^2 = 0.4$, what is the $F$-statistic for the test that the slope is equal to zero?

Solution: The degrees of freedom are $DFM = 1$ and $DFE = n - 2 = 20$. The $F$-statistic is

$$F = \frac{MSM}{MSE} = \frac{SSM/DFM}{SSE/DFE} = \frac{SSM}{SSE} \frac{DFE}{DFM} = \frac{R^2}{(1 - R^2)} \frac{20}{1} = \frac{0.4 \times 20}{0.6} = 13.33.$$ 

The $\alpha = 0.05$ critical value for $F_{1,20}$ is 4.35 (from page 1322), and the $p$-value using that $df$ is 0.0016. We reject $H_0 : \beta_1 = 0$ and conclude that the slope it not zero.

4. Calculate power for the slope using the results of text Problem 1.22 as follows. Assume $n = 16$, $\sigma^2 = MSE$, and $SS_X = 1280$.

(a) Find the power for rejecting the null hypothesis that the regression slope is zero using an $\alpha = 0.02$ significance test when the alternative is $\beta_1 = 0.6$.

Solution: The power against $H_A : \beta_1 = 0.6$ (calculated using SAS) is 1, see Figure 2.

(b) Plot the power as a function of $\beta_1$ for values of $\beta_1$ between -2.8 and +2.8 in increments of 0.2.

Solution: See the attached graph (Figure 3).

The next 5 problems continue the analysis of the plastic hardness data begun in the first homework.
Figure 1: Graph for Problem 2.22

Figure 2: Table for Problem 4a
5. Plot the data using `proc gplot`. Use `frame` as an option with the `plot` statement and include a smoothed function on the plot by using the `i = smnn` option on the `symbol1` statement, where `nn` is a number between 1 and 99. Please use the number 70. Is the relationship approximately linear?

   **Solution:** Yes, the relationship is reasonably linear. (See Figure 4.) There is some slight curvature at lower values of time but nothing substantial.

6. Plot the 94% bounds (confidence band) for the mean (use `i=rlclm` on the `symbol1` statement).

   **Solution:** See the attached graph (Figure 5).

7. Plot the 94% bounds for individual observations (using `i=rlcli`).

   **Solution:** See the attached graph (Figure 6).

8. Give an estimate of the mean hardness that you would expect after 36 and 43 hours; and a 94% confidence interval for each estimate. Which confidence interval is wider and why is it wider?

   **Solution:** Based on the SAS output, the predicted value of hardness at 36 hours is 241.8375, and a 94% confidence interval for the mean hardness is [239.6180, 244.0570]. The SAS output also gives that at 43 hours, the predicted hardness is 256.0781, and
a 94% confidence interval for mean hardness is [252.8479, 259.3083]. The confidence interval for $X = 43$ is wider because the value 43 is farther away from the sample mean $\bar{X}$ than is 36. As a result, the standard error for the prediction is larger.

9. Give a prediction for the hardness that you would expect for an individual piece of plastic after 43 hours; give a 94% prediction interval for this quantity.

**Solution:** We predict a hardness of 256.0781 after 43 hours; we are 94% confident that the hardness value will fall in the interval [248.7144, 263.4418].
Figure 5: Graph for Problem 6

Figure 6: Graph for Problem 7
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Figure 7: Graph for Problem 8