1. Consider the following SAS output giving 5 confidence intervals for the mean of $Y$. If you wanted to guarantee that joint coverage of the five confidence intervals was at least 94%, what confidence level would you use when forming each interval, using the Bonferroni correction? Compute this adjusted confidence interval for the mean of $Y$ when $X = 3$. (Note that some observations have been omitted from the output.)

```
Analysis of Variance

Source DF  Sum of Squares  Mean Square  F Value  Pr > F
Model      1       16183     16183        805.62   <.0001
Error      16      321.39597  20.08725
Corrected Total 17      16504

Root MSE 4.48188  R-Square 0.9805
Dependent Mean 64.00000  Adj R-Sq 0.9793
Coeff Var 7.00294

Parameter Estimates

Variable DF Estimate Standard Error t Value Pr > |t|
Intercept 1 -2.32215 2.56435 -0.91 0.3786
x 1 14.73826 0.51926 28.38 <.0001

Output Statistics

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<th>Obs</th>
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```
2. Based on the following small data set, construct the design matrix, $X$, its transpose $X'$, and the matrices $X'X$, $(X'X)^{-1}$, $X'Y$, and $b = (X'X)^{-1}X'Y$. (Chapter 5 in the book discusses finding the inverse of a matrix.)

\[
X \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{bmatrix} Y \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}
\]

\[
\begin{bmatrix} -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 8 \\ 7 \\ 11 \\ 9 \end{bmatrix}
\]

For the following 5 problems, consider the data given in the file CH06PR18.DAT, which describes a data set ($n = 24$) used to evaluate the relation between intermediate and senior level annual salaries of bachelor’s and master’s level mathematicians ($Y$, in thousand dollars) and an index of work quality ($X_1$), number of years of experience ($X_2$), and an index of publication success ($X_3$).

3. Run the multiple linear regression with quality, experience, and publications as the explanatory variables and salary as the response variable. Summarize the regression results by giving the fitted regression equation, the value of $R^2$, and the results of the significance test for the null hypothesis that the three regression coefficients for the explanatory variables are all zero (give null and alternative hypotheses, test statistic with degrees of freedom, $p$-value, and brief conclusion in words).

4. Give 94% confidence intervals (do not use a Bonferroni correction) for regression coefficients of quality, experience, and publications based on the multiple regression. Describe the results of the hypothesis tests for the individual regression coefficients (give null and alternative hypotheses, test statistic with degrees of freedom, $p$-value, and a brief conclusion in words). What is the relationship between these results and the confidence intervals?

5. Plot the residuals versus the predicted salary and each of the explanatory variables (i.e., 4 residual plots). Are there any unusual patterns?

6. Examine the assumption of normality for the residuals using a qqplot and histogram. State your conclusions.

7. Predict the salary for a mathematician with quality index equal to 5.6, 12 years of experience, and publication index equal to 6.2. Provide a 94% prediction interval with your prediction.