Multi-Period Corporate Default Prediction With Stochastic Covariates

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Outline

Introduction

The Model

Empirical Analysis

The Out-Sample Performance

Extensions and Conclusion
Introduction

The paper provides maximum likelihood estimators of term structures of conditional probabilities of corporate default, incorporating the dynamics of firm-specific and macroeconomic covariates.

Double-stochastic formulation of point process for default and other forms of exit

\[ \lambda_t = \Lambda(X_t) \]

\[ \Lambda(\cdot) \] same across different firms.

Exit for other reasons with an intensity \[ \alpha_t = A(X_t) \]

\[ X_t \] is a Markov state vector of firm-specific and macroeconomic covariates

Exploit time-series dynamics of the explanatory covariates

Given the path of the state-vector process \[ X_t \], the default and other exit times of different firms are conditional independent.
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Remark

• The shape of the term structure of conditional default probabilities reflects the time-series behavior of the covariates, especially leverage targeting by firms and mean reversion in macroeconomic performance.

• However there is no way to check the accuracy of the term structure, and of course no way for calibration directly.

• The double-stochastic setting tends to underestimate the default correlations between firms. The default of one firm has:
  • Direct impact on default intensity
  • Indirect impact from affecting the dependence of intensity on corvariates for another firm.

• The out-of-sample predictive performance of the model is an improvement over that of other available models in the sense of the ability to sort firms according to estimated default likelihoods at various time horizons.
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Set-up

- A probability space \((\Omega, \mathcal{F}, P)\) and an information filtration \(\{\mathcal{G}_t : t \geq 0\}\).
- \(X = \{X_t : t \geq 0\}\) a time homogeneous Markov process in \(\mathbb{R}^d\).
- \((M, N)\) a double-stochastic non-explosive two-dimensional counting process driven by \(X\), with intensities \(\alpha = \{\alpha_t = A(X_t)\}\) for \(M\) and \(\lambda = \{\lambda_t = \Lambda(X_t)\}\) for \(N\).
- Conditional on the path of \(X\), the counting process \(M\) and \(N\) are independent Poisson processes with conditionally deterministic time-varying intensities, \(\alpha\) and \(\lambda\).
- \(\tau = \inf\{t : M_t + N_t > 0\}\)
- \(X_t = (U_t, Y_t)\), where \(U_t\) is firm-specific and \(Y_t\) is macroeconomic.
- \(\mathcal{F}_t = \sigma\left\{(U_s, M_s, N_s) : s \leq \min(t, \tau)\right\} \cup \{Y_s : s \leq t\}\).
- Default time \(T = \inf\{t : N_t > 0, M_t = 0\}\)
Conditional Survival and Default Probabilities

Theorem

On the event \( \{ \tau > t \} \) of survival to \( t \), the \( \mathcal{F}_t \)-conditional probability of survival to time \( t + s \) is

\[
P(\tau > t + s | \mathcal{F}_t) = p(X_t, s) = E\left( e^{-\int_t^{t+s}(\lambda(u) + \alpha(u))du} | X_t \right)
\]

and the \( \mathcal{F}_t \)-conditional probability of default by \( t + s \) is

\[
P(T < t+s | \mathcal{F}_t) = q(X_t, s) = E\left( \int_t^{t+s} e^{-\int_t^z(\lambda(u) + \alpha(u))du} \lambda(z)dz | X_t \right)
\]
More notations

Consider n firms:

- Let $S_i = \inf\{t : M_{it} > 0, N_{it} = 0\}$ and $\tau_i = \min(S_i, T_i)$.
- $X_{it} = (U_{it}, Y_t)$
- $X_{it} = X_{i,k(t)} = Z_{i,k(t)}$, where $k(t)$ denotes the last discrete time-period before $t$, and $Z_i$ is the time homogeneous discrete-time Markov process of covariates for firm $i$.
- Let $\Lambda(X_{it}, \beta)$ denote the default intensity of firm $i$, where $\beta$ is a parameter vector, common to all firms, to be estimated.
- The econometrician’s information set $\mathcal{F}_t$ at time $t$ is

$$\mathcal{I}_t = \{Y_s : s \leq t\} \bigcup \mathcal{I}_{1t} \bigcup \mathcal{I}_{2t} \cdots \bigcup \mathcal{I}_{nt}$$

- $\mathcal{J}_{it} = \{(1_{S_i < u}, 1_{T_i < u}, U_{iu}) : t_{i}^{0} \leq u \leq \min(S_i, T_i, t)\}$ is the information set for firm $i$.
- Conditional on the current combined covariate vector $Z_k = (Z_{1k}, \ldots, Z_{nk})$, $Z_{k+1}$ has a joint density $f(\cdot | Z_k; \gamma)$ for some parameter $\gamma$. 
\[ \mathcal{L}(I_t; \gamma, \beta) = \mathcal{L}(\tilde{U}, Y, t; \gamma) \times \mathcal{L}(S(t), T(t); Y, \tilde{U}, \beta) \]

Thus we can decompose the overall maximum likelihood estimation problem into the separate problems.
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The sample data contains 2,770 firms from 1980 to 2004. The exit types are:

<table>
<thead>
<tr>
<th>Exit type</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>bankruptcy</td>
<td>175</td>
</tr>
<tr>
<td>default</td>
<td>495</td>
</tr>
<tr>
<td>failure</td>
<td>497</td>
</tr>
<tr>
<td>merger-acquisition</td>
<td>872</td>
</tr>
<tr>
<td>other</td>
<td>877</td>
</tr>
</tbody>
</table>

Remark: "Default" includes "bankruptcy" and "failure" includes "default".
Covariates

- The firm’s distance to default

\[ D_t = \frac{\ln \left( \frac{V_t}{L_t} \right) + (\mu_A - \frac{1}{2} \sigma_A^2)T}{\sigma_A \sqrt{T}} \]

- The firm’s trailing 1-year stock return
- The 3-month Treasury bill rate (in percent).
- The trailing 1-year return on the S&P 500 index.

A number of additional covariates are rejected for lack of significance: the U.S 10-year treasury yield, U.S. personal income growth, U.S. GDP growth rate, average Aaa-to-Baa bond yield spread, the firm’s size, and the industry-average distance to default.
Covariate Time-Series Model

For the 3-month and 10-year treasury rates, \( r_{1t} \) and \( r_{2t} \), the model is

\[
r_{t+1} = r_t + k_r (\theta_r - r_t) + C_r \epsilon_{t+1}
\]

where \( C_r \) is a 2 × 2 lower-triangular matrix.

For the distance to default \( D_{it} \) and the log-assets \( V_{it} \) of firm \( i \), and the trailing one-year S&P500 return \( S_t \), we have

\[
\begin{bmatrix}
D_{i,t+1} \\
V_{i,t+1}
\end{bmatrix}
= \begin{bmatrix}
D_{it} \\
V_{it}
\end{bmatrix}
+ \begin{bmatrix}
k_D & 0 \\
0 & k_V
\end{bmatrix}
\begin{bmatrix}
\theta_{i,D} \\
\theta_{i,V}
\end{bmatrix}
- \begin{bmatrix}
D_{it} \\
V_{it}
\end{bmatrix}
+ \begin{bmatrix}
b \cdot (\theta_r - r_t) \\
0
\end{bmatrix}
+ \begin{bmatrix}
\sigma_D & 0 \\
0 & \sigma_V
\end{bmatrix} \eta_{i,t+1}
\]

\[
S_{t+1} = S_t + k_S (\theta_S - S_t) + \epsilon_{t+1}
\]

where

\[
\eta_{it} = Az_{it} + Bw_t
\]

\[
\epsilon_t = \alpha_S u_t + \gamma_S w_t
\]
Default Intensity: Proportional-hazards

\[
\Lambda(x; \mu) = e^{\mu_0 + \mu_1 x_1 + \cdots + \mu_n x_n}
\]

The estimation results are as below:

<table>
<thead>
<tr>
<th>Exit type</th>
<th>constant</th>
<th>DTD</th>
<th>return</th>
<th>3-mo. r</th>
<th>SPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>bankruptcy</td>
<td>-3.099</td>
<td>-1.089</td>
<td>-0.930</td>
<td>-0.153</td>
<td>1.074</td>
</tr>
<tr>
<td></td>
<td>(0.198)</td>
<td>(0.062)</td>
<td>(0.141)</td>
<td>(0.037)</td>
<td>(0.489)</td>
</tr>
<tr>
<td>default</td>
<td>-2.156</td>
<td>-1.129</td>
<td>-0.694</td>
<td>-0.105</td>
<td>1.203</td>
</tr>
<tr>
<td></td>
<td>(0.113)</td>
<td>(0.036)</td>
<td>(0.075)</td>
<td>(0.021)</td>
<td>(0.289)</td>
</tr>
<tr>
<td>failure</td>
<td>-2.148</td>
<td>-1.129</td>
<td>-0.692</td>
<td>-0.106</td>
<td>1.185</td>
</tr>
<tr>
<td></td>
<td>(0.113)</td>
<td>(0.036)</td>
<td>(0.074)</td>
<td>(0.021)</td>
<td>(0.289)</td>
</tr>
<tr>
<td>merger</td>
<td>-3.220</td>
<td>0.021</td>
<td>0.310</td>
<td>-0.137</td>
<td>1.442</td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td>(0.013)</td>
<td>(0.050)</td>
<td>(0.014)</td>
<td>(0.241)</td>
</tr>
<tr>
<td>other</td>
<td>-2.773</td>
<td>-0.072</td>
<td>0.677</td>
<td>-0.167</td>
<td>0.674</td>
</tr>
<tr>
<td></td>
<td>(0.095)</td>
<td>(0.014)</td>
<td>(0.040)</td>
<td>(0.015)</td>
<td>(0.231)</td>
</tr>
</tbody>
</table>

It is surprising that the coefficient for SPX is positive.
Targeted distance to default $\theta_{iD}$

The standard errors of the estimates of $\theta_{iD}$ are responsible for a significant contribution of the standard errors of estimated term structures of default probabilities.
Distance to default dominates the other covariates and Figure 1 indicates that the exponential dependence is at least reasonable for this crucial variable.
Xerox’s distance to default, 0.95, was well below its estimated target, $\theta_{i,D} = 4.4$. And other indications that the company was in significant financial distress at the point were its 5-year default swap rate of 980 basis points and its trailing 1-year stock return of -71%.
Distance to default dominates other Corvaribles

![Graph 1: Annualized Xerox default hazard rates as of January 1, 2000 (solid curve), and with distance to default at one standard deviation (1.33) below its current level of 0.95 (dotted curve), and with distance to default at one standard deviation above current level (dashed curve). The trailing S& P 500 return was −8.6%, the trailing one-year stock return of Xerox was −71%, the 3-month treasury rate was 5.8%, and the 10-year treasury yield was 5.2%.]

![Graph 2: Annualized Xerox default hazard rates as of January 1, 2000 (solid curve), and with the 3-month treasury rate at one standard deviation (3.6%) below the current level of 5.8% (dotted curve), and at one standard deviation above the current level (dashed curve).]
Other empirical results

Figure 6: Annualized Xerox default hazard rates as of January 1, 2004 (solid curve), and with distance to default at one standard deviation (1.33) below its current level of 3.7 (dotted curve), and with distance to default at one standard deviation above current level (dashed curve).

Figure 7: Estimated conditional density of Xerox’s default time as of January 1, 2001. Bottom plot: the estimated default time density, incorporating the impact of survival from merger and acquisition. Top plot: the estimated default-time density obtained by ignoring (setting to zero) the intensity of other exits.
Out of sample performance means the ability of the model to sort firms according to estimated default likelihoods at various time horizons. The one-year-ahead out-of-sample accuracy ratio for default prediction of the model is 88% while the one based on Moody’s credit ratings is 65%, those based on ratings adjustments for placements on Watchlist and Outlook are 69% and those based on sorting firms by bond-yield spreads average 74%.
Accuracy ratios for different exit types


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Underestimate the default correlation

There are literatures concentrating on the analysis of default correlation variations:

• Sanjiv R. Das, Laurence Freed, Gary Geng, Nikunj Kapadia (2002), *Correlated Default Risk*. Instead of discussing time-series structure of corvariates, the paper directly analyzed the time-series structure of the vector intensities. Finally the default intensities and correlations can be modeled jointly in a regime-shifting framework.

• Sanjiv R. Das, Darrell Duffie, Nikunj Kapadia, and Leandro Saita (2007)
  Doubly stochastic assumption under which firm’s default times are correlated only as implied by the correlation of factors determining their default intensities was tested. Using data on U.S. corporations from 1979 to 2004, this assumption is violated in the presence of contagion or "frailty" (unobservable explanatory variables that are correlated across firms). The tests do not depend on the time-series properties of default intensities. They find some evidence of default clustering exceeding that implied by the doubly stochastic model with the given intensities.
Test of Default Intensity Conditional Independence

David Lando, Mads Stenbo Nielsen (2009), *Correlation in corporate defaults: Contagion or conditional independence?*

The paper revisited a test for conditional independence in intensity models of default proposed by Das, Duffie, Kapadia, and Saita (2007) (DDKS). Based on a sample of US corporate defaults, they reject the conditional independence assumption but also observe that the test is a joint test of the specification of the default intensity of individual firms and the assumption of conditional independence. This paper showed that using a different specification of the default intensity, and using the same test as DDKS, you cannot reject the assumption of conditional independence for default histories recorded by Moody’s in the period from 1982 to 2006. The paper also showed, that the test proposed by DDKS is not able to detect all violations of conditional independence. Specifically, the tests will not capture contagion effects which are spread through the explanatory variables used as conditioning variables in the Cox regression and which determine the default intensities of individual firms. The authors therefore perform different tests to see if firm-specific variables, i.e quick ratios and distance-to-default, are affected by defaults. They find no influence from defaults on Quick ratios, but some influence on distance-to-default. This suggests, that violations of conditional independence do indeed arise from balance sheet effects.
Thank You!