INTRODUCTION TO PROBABILITY MODELS

Lecture 13

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REVISION

- **Permutation:** Ordered arrangement of \( r \) distinct objects from a set of \( n \) objects.

\[
_{n}P_{r} = P_{r}^{n} = \frac{n!}{(n - r)!}
\]

- **Combination:** Unordered arrangement of \( r \) distinct objects from a set of \( n \) objects.

\[
_{n}C_{r} = C_{r}^{n} = \frac{n!}{(n - r)!r!}
\]

- **Multinomial Coefficient:** \( m \) objects are in \( k \) distinct groups, size of groups are \( m_1, m_2, \ldots, m_k \), number of ways to order these are:

\[
\binom{m}{m_1, m_2, \ldots, m_k} = \frac{m!}{m_1!m_2!\cdots m_k!}
\]
PROPERTIES OF EXPECTED VALUE

X, Y are random variables, c and d are constant

- $E[c] = c$
- $E[cX] = cE[X]$
- $E[X + Y] = E[X] + E[Y]$
- $E[cX + dY] = cE[X] + dE[Y]$
PROPERTIES OF VARIANCE

X, Y are random variables, c and d are constant

- $\text{Var}(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2$
- $\text{Var}(c) = 0$
- $\text{Var}(cX) = c^2 \text{Var}(X)$
- If X and Y are independent,
  $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$
- If X and Y are independent,
  $\text{Var}(cX + dY) = c^2 \text{Var}(X) + d^2 \text{Var}(Y)$
EXAMPLE 1

Suppose $X$ and $Y$ are random variables with $E[X] = 3, E[Y] = 4$ and $Var(X) = 2, Var(Y) = 1$. Find

1. $E[2X + 1]$
2. $E[X - Y]$
3. $E[X^2]$
4. $E[X^2 - 4]$
5. $E[(X - 4)^2]$
6. $Var(2X - 4Y)$