INTRODUCTION TO PROBABILITY MODELS

Lecture 26

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REMINDER

1. Homework 4 is due on Oct 30
2. Exam 2 will be from 8:00pm to 9:30pm on Thursday, November 2nd
EXAMPLE 1

At a high school track and field tournament, Mark’s high jumps vary evenly from 1.8 meters to 2.15 meters, while Dan’s high jumps vary evenly from 1.75 to 2.3 meters.

1. Let $M$ be the length of one of Mark’s high jumps. What are the distribution and parameter(s) of $M$?
2. What is that the probability that Mark jumps between 1.88 and 2.05 meters?
3. Which jumper’s jumps has the smaller standard deviation?
4. What is the probability that one of Dan’s high jumps is exactly 2.0 meters?
5. What length cuts off the highest 25% of Dan’s high jumps?
TIME FOR QUIZ
EXPONENTIAL RANDOM VARIABLE

- **The definition of** $X$: The waiting time until the first success
- **Support**: $X \in [0, +\infty)$ or $X \geq 0$
- **Parameter**: $\mu$, the average amount of time for one success
- **PDF**: $f_X(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$, for $x \geq 0$
- **CDF**:
  
  $F_X(x) = P(X \leq x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\frac{x}{\mu}}, & x \geq 0 \end{cases}$

- **Expected Value**: $E[X] = \mu$
- **Variance**: $Var(X) = \mu^2$
- **Notation**: $X \sim \text{Exp}(\mu)$ or $X \sim \text{Exponential}(\mu)$
IMPORTANT PROPERTIES FOR THE EXPONENTIAL DISTRIBUTION

If $X \sim \text{Exp}(\mu)$

- Tail Probability formula: $P(X > x) = e^{-\frac{x}{\mu}}$
- Memoryless Property:
  
  $P(X > s + t|X > s) = P(X > t)$
EXAMPLE 2

It is your birthday and you are waiting for someone to write you a birthday message on Facebook. On average (on your birthday) you receive a facebook message every 10 minutes. Assume that birthday messages arrive independently.

1. What is the probability the next posting takes 15 minutes or longer to appear? What distribution, parameter(s) and support are you using?
2. What is the standard deviation of the time between birthday postings?
3. What is the probability that it takes 12.5 minutes for the next birthday posting?
4. Suppose that the most recent birthday posting was done at 1:40 pm and it is now 1:45 pm. What is the probability that you will have to wait until 1:53 pm or later for the next message?
5. What is the probability that your wait time for the next three messages is less than 8 minutes?
6. What is your median waiting time for birthday messages?