INTRODUCTION TO PROBABILITY MODELS

Lecture 18

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REMINDERS

1. The fourth quiz will be on this **Wednesday, Oct 4**
2. The third homework is posted and due on **Oct 16**
3. No class on **Oct 6** as Exam 1 compensation


**EXAMPLE 1**

Consider a game where we will roll a fair die. We will roll it until we get a 5. What is the probability that it will take 3 rolls to get the 5?

Think about:

- Are the subsequent rolls independent?
- What about the probability of success on each roll?
- Do we have a set number of trials?
GEOMETRIC RANDOM VARIABLE
CHARACTERISTICS OF THE GEOMETRIC DISTRIBUTION

- **The definition of** $X$: the number of trials to get the first success
- **Support**: $\{1, 2, \ldots\}$, **NOTE**: **NO ZERO**!
- **Parameter**: $p$, the probability of success in one trial
- **PMF**: $P_x(x) = p(1 - p)^{x-1}$
- **Expected Value**: $E[X] = \frac{1}{p}$
- **Variance**: $Var(X) = \frac{1-p}{p^2}$
- $X \sim Geom(p)$
EXAMPLE 2

Suppose Dunphy is really bad at tossing a Frisbee and unfortunate hits people walking by at a rate of 1 out of every 5 people.

1. What is the probability that his first accidental hitting is the $6_{th}$ or $7_{th}$ person to walk by?
2. What is the probability that more than 7 people walk past before he hits one with the Frisbee?
IMPORTANT PROPERTIES FOR THE GEOMETRIC DISTRIBUTION

- Tail Probability formula: $P(X > k) = (1 - p)^k$
- Memoryless Property:
  
  $P(X > s + t | X > s) = P(X > t)$ and
  
  $P(X < s + t | X > s) = P(X < t)$
EXAMPLE 2 CONTINUED

3. Four people have walked past Dunphy and none have been hit by a Frisbee. What is the probability that at most 9 walk by until the first person is hit by a Frisbee?

4. Four people have walked past Dunphy and none have been hit by a Frisbee. What is the probability that at least 10 walk by until the first person is hit by a Frisbee?
EXAMPLE 3

Shaq is shooting free throws in the gym. He intends to stay until he makes one. His probability of making one on any free throw is 0.527. Let X be the number of attempts until he makes one.

1. Distribution, parameter, support?
2. Expected number of shots until he makes one?
3. Probability he makes his first shot on the $4_{th}$ try?
4. Probability it takes him at least 4 shots to make $1_{st}$?
5. Probability it takes him exactly 4 shots if he already missed the first?
6. Probability it takes him at least 4 shots if he already missed the first?