INTRODUCTION TO PROBABILITY MODELS

Lecture 13

Qi Wang, Department of Statistics

Sep 20, 2017
REVISION

• **Permutation:** Ordered arrangement of r distinct objects from a set of n objects.

\[ nP_r = P^n_r = \frac{n!}{(n - r)!} \]

• **Combination:** Unordered arrangement of r distinct objects from a set of n objects.

\[ nC_r = C^n_r = \frac{n!}{(n - r)!r!} \]

• **Multinomial Coefficient:** m objects are in k distinct groups, size of groups are \( m_1, m_2, \ldots, m_k \), number of ways to order these are:

\[ \binom{m}{m_1, m_2, \ldots, m_k} = \frac{m!}{m_1!m_2! \cdots m_k!} \]
TIME FOR QUIZ
PROPERTIES OF EXPECTED VALUE

X, Y are random variables, c and d are constant

- $E[c] = c$
- $E[cX] = cE[X]$
- $E[X + Y] = E[X] + E[Y]$
- $E[cX + dY] = cE[X] + dE[Y]$
PROPERTIES OF VARIANCE

X, Y are random variables, c and d are constant

- $\text{Var}(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2$
- $\text{Var}(c) = 0$
- $\text{Var}(cX) = c^2 \text{Var}(X)$
- If X and Y are independent,
  $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$
- If X and Y are independent,
  $\text{Var}(cX + dY) = c^2 \text{Var}(X) + d^2 \text{Var}(Y)$
EXAMPLE 1

Suppose $X$ and $Y$ are random variables with $E[X] = 3$, $E[Y] = 4$ and $Var(X) = 2$. Find

1. $E[2X + 1]$
2. $E[X - Y]$
3. $E[X^2]$
4. $E[X^2 - 4]$
5. $E[(X - 4)^2]$
6. $Var(2X - 4Y)$