MA/STAT 639.
Stochastic Processes II.

Spring 2005.

Prof. Frederi Viens

Classroom and time:
Tu 5:30-8:00pm in UNIV 219

I can't believe they gave us a classroom in University Hall this late in the evening...

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Office hours: Tu 1:30 - 3:00 in MATH 504.

"Required" textbook.

- Continuous Martingales and Brownian Motion. Daniel Revuz, Marc Yor. 3rd edition. Springer Verlag, 1998.

Recommended reading.

- Brownian Motion and Stochastic Calculus. Ioannis Karatzas, S. E. Shreve. 2nd edition, paperback. Springer Verlag, 1998.
- An Introduction to Analysis on Wiener Space. Ali-Suleyman Üstünel. (Lecture notes in mathematics **1610**). Springer-Verlag,1995.
- The Malliavin Calculus and Related Topics. David Nualart. Springer-Verlag,1995.
- The instructor will also recommend a number of recent research articles in stochastic analysis.

Prerequisite: An excellent level in graduate probability theory (e.g. MA/STAT 538 and/or MA/STAT 539). IT IS NOT NECESSARY TO HAVE TAKEN MA/STAT 638!

Audience: People who want to write, or are writing, a Ph.D. dissertation in/using Probability Theory (Math or Stat departments).

Course description. The goal is threefold:

to understand the big picture of how the mathematical theory of Stochastic Processes can be built around the canonical example of Brownian motion;

to know the proofs and solve the exercise problems at an arbitrary level of difficulty in the first two texts listed above; in particular, the first text listed is commonly agreed to contain the best-thought-out and most ingenious list of problems, with an excellent connection/flow to the body of the text, a text which is used by the most talented probability students in Europe, while the second-listed text is generally quoted as the bible by North American students of Stochastics.

to introduce students to various topics of great contemporary interes in stochastic analysis, which cannot be found in standard textbooks, including questions related to analysis on Wiener space, fractional Brownian motion, stochastic PDEs, and more.

The second page of this syllabus contains a list of topics that we hope to cover, roughly in the order presented below, with possible omissions and additions.

Grading scheme.

Since the course has NO grader, I am still debating what kinds of assignments will be required to monitor the progress and understanding of each student. This will be discussed in class as the semester progresses.

Tentative outline of MA/STAT 638

- Stochastic Differential Equations

  Review of Existence and Uniqueness in the Case of Lipschitz Coefficients

  Generators, Diffusions and Ito Processes, a review of Girsanov's Theorem, and Applications to Weak Solutions

  The Case of Holder Coefficients in Dimension One

  Connection with partial differential equations: Feynman-Kač formulae

  Connection with partial differential equations: Dirichlet problem

  Stochastic partial differential equations

- Fractional Brownian motion

  Definition and canonical Hilbert spaces

  Skorohod integration via the Malliavin calculus: a new definition

  Pathwise integration of Russo-Vallois

- Local Times

  The Local Time of Brownian Motion

  Ito-Tanaka formula

  Local time for fractional Brownian motion

  Levy's theory of Brownian local time

- Regularity of Gaussian processes

  General theory; Borell-type inequalities; Dudley-Fernique theorems

  Applications to stochastic PDEs

  Subgaussian theory and Ustunel's analysis on Wiener space

  Possible applications to local time regularity

- More applications of the Malliavin calculus

  Lyapunov exponents for stochastic PDEs

  Anticipating stochastic calculus

- Levy Processes and Excursions

FYI, last semester's class MA/STAT 638 contained:

- Introduction. Construction of Brownian motion, its path properties; stopping times
- Martingales: Maximal Inequalities and Applications; Optional Stopping Theorem
- Markov and Strong Markov Processes
- Stochastic Integration: Ito formula; Burkholder-Davis-Gundy Inequalities
- Girsanov theorem
- Introduction to Strong Solutions of Stochastic Differential Equations.