MA/STAT 638. Stochastic Processes I.

Fall 2004. Frederi Viens, Associate Professor of Statistics and Mathematics.
Classroom and times: Tu Th 10:30-11:45 in UNIV 201

Telephone: (765) 494-6035
E-mail: viens@purdue.edu
Office hours: Tu 1:00 - 3:00 and Th 9:30-10:30, in MATH 504.

Recommended textbooks.

- Continuous Martingales and Brownian Motion. Daniel Revuz, M. Yor. 3rd edition. Springer Verlag., 1998. ISBN: 3540643257.
- Brownian Motion and Stochastic Calculus. Ioannis Karatzas, S. E. Shreve. 2nd edition, paperback. Springer Verlag, 1998. ISBN: 3540976558.
- Diffusions, Markov Processes and Martingales: Foundations (Vol. 1), and Itô Calculus (Vol. 2). L. C. Rogers, David Williams. 2nd edition, paperback. Cambridge U.P., 2000. ISBN: 0521775949 and ISBN: 0521775930.

Prerequisite: An excellent level in graduate probability theory (e.g. MA/STAT 538 and/or MA/STAT 539).

Audience: People who want to write, or are writing, a Ph.D. dissertation in/using Probability Theory (Math or Stat departments); also, people who need to understand probability theory at a high level for a Ph.D. dissertation in other fields (e.g. Signal Processing in Electrical Engineering; Quantitative methods in Industrial Engineering, Economics, or Finance; Turbulence in Atmospheric Science).

Course description. The goal is twofold:

to understand the big picture of how the mathematical theory of Stochastic Processes can be built around the canonical example of Brownian motion;

to know the proofs and solve the exercise problems at an arbitrary level of difficulty in the first two texts listed above; in particular, the first text listed is commonly agreed to contain the best-thought-out and most ingenious list of problems, with an excellent connection/flow to the body of the text, a text which is used by the most talented probability students in Europe, while the second-listed text is generally quoted as the bible by North American students of Stochastics.

The second page of this syllabus contains a list of topics that we hope to cover, roughly in the order presented below, with possible omissions and additions. It is certain that the complete list below is sufficient for two semesters. Consequently, one can expect that many of the topics will be covered only in the Spring semester, as MATH/STAT 639.

Grading scheme.

Since the course has a grader, there will be a certain number of graded homework assignments and in-class quizzes, whose **tentative** relative importance in the final grade would be 20% for the homework and 40% for the quizzes.

Final exam: 40%. It will either be an in-class individual written exam, or an individual oral exam, or a group project including an in-class presentation. Most likely, individual student will be able to choose between any one of these three options.

Tentative outline of MA/STAT 638

- Introduction. Construction of Brownian motion.

  Examples of Stochastic Processes.

  Construction of Brownian Motion.

  Gaussian Processes

  Local Properties of Brownian Paths

  Filtrations and Stopping Times

- Martingales

  Maximal Inequalities and Applications

  Convergence and Regularization Theorems

  Optional Stopping Theorem

- Markov Processes

  Definitions

  Strong Markov Property

- Stochastic Integration

  Quadratic Variations

  Ito Stochastic Integral

  Ito's Formula and First Applications

  Burkholder-Davis-Gundy Inequalities

- Representation of Martingales

  Continuous Martingales as Time-changed Brownian Motions

  Conformal Martingales and Planar Brownian Motion

  Brownian Martingales

  Integral Representations

- Fractional Brownian motion

  Definition and canonical Hilbert spaces

  Skorohod integration via the Malliavin calculus

  Pathwise integration of Russo-Vallois

- Generators and Time Reversal

  Infinitesimal Generators

  Diffusions and Ito Processes

  Time Reversal and Applications

- Girsanov's Theorem and First Applications

  Girsanov's Theorem

  Application of Girsanov's Theorem to the Study of Wiener's Space

  Functionals and Transformations of Diffusion Processes

- Stochastic Differential Equations

  Formal Definitions

  Existence and Uniqueness in the Case of Lipschitz Coefficients

  Weak solutions

  The Case of Holder Coefficients in Dimension One

  Connection with partial differential equations: Feynman-Kač formulae

  Connection with partial differential equations: Dirichlet problem

  Stochastic partial differential equations

- Local Times

  The Local Time of Brownian Motion

  Ito-Tanaka formula

  Local time for fractional Brownian motion; regularity issues

  Levy's theory of Brownian local time

- Levy Processes and Excursions