MA/STAT 598 F. Mathematics of Finance.
Purdue University, Fall 2002.

Professor Frederi Viens
Classroom and time: MATH 211, Tu Th 9:00 - 10:15.

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Textbook. Arbitrage theory in continuous time. Tomas Bjork. Oxford U.P., 1998.

Suggested additional reading.

- The mathematics of financial derivatives. A student introduction. P. Wilmott, S. Howison, J. Dewynne. Cambridge U.P. 1995. Chapters 11 to 16. *For more detail on analytic methods for american options.*
- Introduction to Stochastic Calculus Applied to Finance. D. Lamberton, B. Lapeyre. Chapman Hall/CRC Press 1996. *For people who want more math than we will be doing...*
- Implementing derivative models. L. Clewlow, Ch. Strickland. John Wiley and Sons, Ltd., 1998. *For people who want to read ahead on the numerical portion of the Spring 2002 course MA/STAT 598 G.*

Prerequisites and suggested preparation.

- A graduate introduction to probability theory (no measure theory needed): MA 519 (or equivalent) strongly desirable; otherwise, concurrent enrollment required.
- Multivariate calculus: MA 261 (or equivalent) required; a higher course desirable.
- Real analysis: MA 440 (or equivalent) required. MA 504 desirable, or concurrent enrollment required.
- Differential equations: MA 360 or 364 or 366 (or equivalent) required.

Concurrent and future suggested enrollment.

- Introduction to Investment Science: MA/STAT 598 Y. *This basic financial engineering course is a very useful, self-taught, group-reading class, which is valuable for fulfilling the requirements for a Math or Stat MS degree with Computational Finance (CF) specialization. Taught in the Fall. Also consider the second semester course MA/STAT 598 Z, taught in the Spring.*
- Financial Engineering: IE 590 A. *A course similar to MA/STAT 598 Y in content, taught in the department of Industrial Engineering, usually in the Spring.*
- Introduction to Futures and options: MGMT 641. *This is an MBA-style class, valuable for fulfilling the requirements for a Math or Stat MS degree with CF specialization, providing sound financial intuition and detailed information. Taught in the Fall, but conflicts with MA/STAT 598F this year.*
- Risk Management Using Derivatives: MGMT 643. *An advanced and demanding management class taught in the Spring by Professor A.C. Sullivan, a specialist in the field. Strongly recommended.*
- Advanced probability and financial options, with numerical methods: MA/STAT 598 G. Spring 2002. *Sequel of MA 598 F. Required for the MS degree with CF specialization. A must if you are serious about learning Financial Mathematics.*
- Econometrics and times series courses taught in the department of Economics by professors Ch. Dahl and N. Swanson.

Course description

We will provide an introduction to the mathematical tools and techniques of modern finance theory, in the context of Black-Scholes-style option pricing. The typical (pricing) question is: how much should you charge someone for allowing them the right to purchase a certain stock from you at a given price and a given time in the future? Once a price has been determined, the most important question is that of hedging: how can you ensure that the price you charge for the option is invested in order to cover your risk no matter what happens to the future stock movements. The typical (Black-Scholes) assumption is that the relative differential of the stock price is proportional to the sum of a constant term (constant interest rate) and a random noise term. Under this assumption, to answer the pricing question, the main mathematical tool is stochastic calculus and its connection to partial differential equations. These mathematics will be the object of a thorough introduction at an elementary level, without measure theory. This toolbox will enable us to derive the main pricing and hedging results, and to treat the many examples and topics outlined below. Towards the end of the semester, we will cover a more difficult topic, that of stochastic portlio optimization: how do you maximize the expected future return of a portfolio using the Black-Scholes model. The main tool here will be stochastic control theory and its associated Hamilton-Jacobi-Bellman equations.

Grading scheme

- Bi-monthly quizzes, averaged: 30%
- Homework assignments, averaged: 10%
- Class project: 30%
- Take-home exam: 30%

Quizzes

Every couple of weeks, a 30-minute (sometimes longer) in-class quiz will test your grasp of the material covered in class in the previous two weeks, although some quizzes may be more comprehensive. You will NOT be allowed to use textbooks, notes, or any other aid during the quizzes. See the note below about plagiarism[1], which will not be tolerated.

Class project

You will be required to turn in one class project/paper and give an in-class presentation based on it. Project topics will be taken from the textbook's latter chapters (Ch 11 to 16) and associated exercises, and/or from the suggested readings and their associated exercises, and/or from students' personal readings (with instructor's accord), and will be due sometime in the last third of the semester. We will identify small groups of students to work together on a single project, and to give a joint presentation.

Homework

Homework problems will be assigned with the reading and collected and graded approximately bimonthly. While it is acceptable to work in groups on homework problems, each student must turn in a separate assignment; identical solutions are NOT acceptable. Your homework must reflect YOUR understanding of the material.

Final exam

It will be a take-home exam, due one week from the day it is assigned; this due date will be near the beginning of finals week, but early completion will be encouraged. Unlike homework assignments, you may not discuss the final exam with anyone; signs of collaboration or plagiarism[1] will be dealt with harshly.

[1] Plagiarism is the act of presenting someone else's work as your own. This includes finding the answer to a given problem in a book, in someone else's assignment, or requesting the answer from someone, and copying from it. Contrary to popular belief, a correct solution to a given mathematical problem is almost never unique, and plagiarism in a mathematical assignment is very easy to detect.

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Outline of the course

- (Bjork Chap. 2). The binomial model (discrete time).
- (Bjork Chap. 3). Stochastic integrals.
- (Bjork Chap. 4). Stochastic differential equations.
- (Bjork Chap. 5 and 6). Black-Scholes pricing.
- (Bjork Chap. 7, 8, 9, and 10). Black-Scholes hedging (completeness, Delta-hedging, and incompleteness).
- (Bjork Chap. 12 and 13; WH&D Chap. 11 to 15) Exotic and path-dependent options.
  - In and out contracts
  - Ladder options
  - Lookback options
  - Asian options
- (WH&D Chap. 16) Transaction costs.
- (Bjork Chap 14). Portfolio optimization.
- (Bjork Chap 15-20 as time allows). Interest rate models.

[for your information] Outline of Spring 2002 course MA/STAT 598 G.

Text: same as Fall course, plus materials from Lamberton-Lapeyre, Wilmott-Howison-Dewynne, Clewlow-Strickland.

Topics:

- Any of the topics above that are not covered in the Fall
- Interest rate models (continued).
- American options and stochastic optimal stopping
- Numerical methods for solving the pricing partial differential equations.
  - Binomial methods
  - Monte-Carlo methods
  - Finite difference methods
  - Special methods for American options
  - Special methods for interest-rate derivatives.