Bayesian inference for Matérn repulsive processes
(Supplementary material)

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1. Supplementary
The plots of figure 1 consider the J-function (van Lieshout and Baddeley, 1996), a measure of spatial interaction with $J(r)$ corresponding to the ratio of the probability there is no event at distance $r$ from another event to the probability there is no event at distance $r$ from an arbitrary point in the space. For a Poisson process, these two probabilities are equal, so that $J(r)$ equals 1. Repulsion correspond to $J(r) > 1$, while $J(r) < 1$ suggests a clustered pattern. The continuous magenta line in figure 1 plots the empirical $J(r)$ as a function of distance $r$ for the pine tree dataset. Again we see that this is a non-Poisson repulsive process. The blue envelope shows posterior predictive estimates for the J-function produced by fitting the Matérn type-III hardcore process (top left), softcore process (top right) and with probabilistic thinning (bottom left). See the main text for a description of these. As with the L-function, the predictive intervals for the hardcore and softcore processes do not fit the data, but contrast the generalized Matérn with probabilistic thinning does a much better job. The bottom right subplot shows predictions from a MLE fit of a Strauss process, which also produces reasonable (but not good) fits.

The top row of figure 2 shows posterior predictive values for a Matérn process with probabilistic thinning for mild (left) and moderate (right) neuropathy. The bottom row shows fits for a Strauss process, again, these are worse.

Figure 3 shows the nonstationary J-function estimated from the Greyhound dataset. This statistic does not capture a significant deviation from Poisson (unlike the L-function), but both the inhomogeneous Matérn model and the inhomogeneous Poisson process fit it well.

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Fig. 1. Posterior predictive values of J-functions for the Swedish pine tree dataset. (Top left) is the Matérn hardcore model, (top right) is the softcore model, and (bottom left) is probabilistic thinning. (Bottom right): predictive values for a Strauss process fit.

Fig. 2. (Top row): Posterior predictive values of the J-functions for the Matérn model with probabilistic thinning for mild (left) and moderate (right) neuropathy. (Bottom row): Corresponding predictive values for Strauss process fits.
Fig. 3. Inhomogeneous J-function for the Greyhound dataset: (left) posterior predictive values for nonstationary Matérn, and (right) fit of an inhomogeneous Poisson process.