1. Problem 7.4.

(a) When \( n = 25 \), the 95% confidence interval of \( \mu \), the mean of stray-load loss, is
\[
[\bar{X} - z_{0.025} \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{0.025} \frac{\sigma}{\sqrt{n}}] = [58.3 - 1.96 \frac{3}{\sqrt{25}}, 58.3 + 1.96 \frac{3}{\sqrt{25}}] = [57.124, 59.476].
\]

(b) When \( n = 100 \), the 95% confidence interval of \( \mu \), the mean of stray-load loss, is
\[
[\bar{X} - z_{0.025} \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{0.025} \frac{\sigma}{\sqrt{n}}] = [58.3 - 1.96 \frac{3}{\sqrt{100}}, 58.3 + 1.96 \frac{3}{\sqrt{100}}] = [57.712, 58.888].
\]

(c) When \( n = 100 \), the 99% confidence interval of \( \mu \), the mean of stray-load loss, is
\[
[\bar{X} - z_{0.005} \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{0.005} \frac{\sigma}{\sqrt{n}}] = [58.3 - 2.576 \frac{3}{\sqrt{100}}, 58.3 + 2.576 \frac{3}{\sqrt{100}}] = [57.527, 59.073].
\]

(d) When \( n = 100 \), the 82% confidence interval of \( \mu \), the mean of stray-load loss, is
\[
[\bar{X} - z_{0.09} \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{0.09} \frac{\sigma}{\sqrt{n}}] = [58.3 - 1.341 \frac{3}{\sqrt{100}}, 58.3 + 1.341 \frac{3}{\sqrt{100}}] = [57.898, 58.702].
\]

(e) The length of the confidence interval is \( 2z_{\alpha/2} \sigma / \sqrt{n} \). If the length is less than 1, then we have
\[
\frac{2z_{\alpha/2} \sigma}{\sqrt{n}} \leq 1.0 \Rightarrow n \geq [2(2.576)(3)]^2 = 238.88.
\]

Thus, we can take \( n = 239 \). \((n = 240 \text{ is OK})\).


(a) (Version 9) The large sample 95% confidence interval is
\[
[\bar{x} - z_{0.025} \frac{s}{\sqrt{n}}, \bar{x} + z_{0.025} \frac{s}{\sqrt{n}}] = [1427 - 1.96 \times \frac{325}{\sqrt{514}}, 1427 + 1.96 \times \frac{325}{\sqrt{514}}] = [1399, 1455].
\]

(Version 8) The large sample 95% confidence interval is
\[
[\bar{x} - z_{0.025} \frac{s}{\sqrt{n}}, \bar{x} + z_{0.025} \frac{s}{\sqrt{n}}] = [89.10 - 1.96 \frac{3.73}{\sqrt{168}}, 89.10 + 1.96 \frac{3.73}{\sqrt{168}}] = [88.54, 89.66].
\]

We say that the average fracture strength has been precisely estimated.

(b) (Version 9) We can roughly assume \( \sigma = 320 \). Then, we need
\[
\frac{2(1.96)(320)}{\sqrt{n}} \leq 50 \Rightarrow n \geq \frac{4(1.96)^2(320)^2}{50^2} = 629.4 \Rightarrow n \geq 630.
\]

Therefore, \( n \) should be at least 630. We choose \( n = 630 \).

(Version 8) If \( \sigma = 4 \) is known, we need
\[
\frac{1.96(4)}{\sqrt{n}} \leq 0.5 \Rightarrow n \geq 245.85.
\]

Then, we take \( n = 246 \).
3. Problem 7.23.

(a) It is known that the 95% confidence interval of $p$ is about

$$-2.576 \leq \frac{X - np}{\sqrt{np(1-p)}} = \frac{24 - 37p}{\sqrt{37p(1-p)}} \leq 2.576 \Rightarrow 0.4385 \leq p \leq 0.8136.$$

(b) The answer

$$n \approx \left( \frac{2.58}{0.05} \right)^2 \approx 2663.$$

I will treat it as the correct answer if the result is close.

4. Problem 7.29.

(a) $\alpha = 1 - 0.95 = 0.05$. The critical value is $t_{2.10} = t_{0.025,10} = 2.228$.

(b) Similarly, $t_{0.025,20} = 2.086$.

(c) $\alpha = 1 - 0.99 = 0.01$. The value is $t_{2.20} = t_{0.005,20} = 2.845$.

(d) Similarly $t_{0.005,50} = 2.678$.

(e) $\alpha = 0.01$. The critical value is $t_{0.25} = 2.485$.

(f) $\alpha = 0.025$, but the critical value is $-t_{0.025,5} = -2.571$.

5. Problem 7.33.

(b) Yes. It looks like a normal distribution.

(c) For this data, we have $n = 17, \bar{x} = 438.29$ and $s^2 = 229.3$. The 95% confidence interval is

$$\bar{x} \pm t_{0.025,16} \frac{s}{\sqrt{n}} = [430, 446.0].$$

6. Problem 7.46(b). Based on the data, we have $n = 15, \bar{x} = 37.61$, and $s^2 = 6.613$. Then, the 95% confidence interval for $\sigma^2$ is

$$\frac{(n - 1)s^2}{\chi_{0.025,14}} \frac{(n - 1)s^2}{\chi_{0.975,14}} = \left[ \frac{14 \times 6.613}{26.12}, \frac{14 \times 6.613}{5.629} \right] = [3.544, 16.447].$$

The 95% confidence interval for $\sigma$ is $[\sqrt{3.544}, \sqrt{16.447}] = [1.883, 4.056]$. 

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