1. Problem 3.29.

(a) 
\[ E(X) = \sum_{k=0}^{4} kp_k = 1(0.05) + 2(0.10) + 4(0.35) + 8(0.40) + 16(0.10) = 6.45. \]

(b) 
\[ V(X) = \sum_{k=0}^{4} (k - 2.06)^2 p_k \]
\[ = (1 - 6.45)^2 0.05 + (2 - 6.45)^2 0.10 + (4 - 6.45)^2 0.35 + (8 - 6.45)^2 0.45 + (16 - 6.45)^2 0.05 \]
\[ = 15.6475. \]

(c) The standard error is 
\[ \sqrt{V(X)} = \sqrt{11.54} = 3.3971. \]

(d) 
\[ E(X^2) = \sum_{k=0}^{4} k^2 p_k = 1(0.05) + 4(0.10) + 16(0.35) + 64(0.40) + 256(0.1) = 57.25. \]

Thus, 
\[ V(X) = E(X^2) - [E(X)]^2 = 57.25 - 6.45^2 = 15.6475. \]


(a) 
\[ E(Y) = 0 \times 0.6 + 1 \times 0.25 + 2 \times 0.1 + 3 \times 0.05 = 0.6. \]

(b) 
\[ E(100Y^2) = 0 \times 0.6 + 100 \times 0.25 + 400 \times 0.1 + 900 \times 0.05 = 110. \]

3. Problem 3.32.

(a) 
\[ E(X) = 16(0.2) + 18(0.5) + 20(0.3) = 18.2. \]
\[ E(X^2) = 16^2(0.2) + 18^2(0.5) + 20^2(0.3) = 333.2. \]
\[ V(X) = 333.2 - 18.2^2 = 1.96. \]

(b) You can do like 
\[ E(70X - 650) = 70E(X) - 650 = 70(18.2) - 650 = 624 \]

or 
\[ E(70X - 650) = 70(16 - 650)(0.2) + 70(18 - 650)(0.5) + 70(20 - 650)(0.3) \]
\[ = 624. \]
(c) You can do like
\[ V(70X - 650) = 70^2V(X) = 70^2(1.96) = 9604, \]
or you can do like
\[
E[(70X - 650)^2] \\
= (70 \times 16 - 650)^2(0.2) + (70 \times 18 - 650)^2(0.5) + (70 \times 20 - 650)^2(0.3) \\
= 3988980.
\]
\[ V(70X - 650) = 398980 - 624^2 = 9640. \]

(d) \[ E(X - 0.008X^2) = E(X) - 0.008E(X^2) = 18.2 - 0.008(333.2) = 15.534. \]

\[ E(X) = 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3 + 4 \times 0.1 = 2.3. \]

and
\[ E(X^2) = 1^2 \times 0.2 + 2^2 \times 0.4 + 3^2 \times 0.3 + 4^2 \times 0.1 = 6.1. \]

Thus,
\[ V(X) = E(X^2) - E^2(X) = 6.1 - 2.3^2 = 0.81. \]

Then, the pound left is \( Y = 100 - 5X \). Thus, we have
\[ E(Y) = E(100 - 5X) = 100 - 5E(X) = 88.5 \]

and
\[ V(Y) = V(100 - 5Y) = 25V(Y) = 20.25. \]

5. Problem 3.41.
Let \( h(X) = aX + b \), \( E(X) = \mu \) and \( V(X) = \sigma^2 \). Then
\[ E[h(X)] = E[aX + b] = a\mu + b. \]

Thus
\[ V(X) = E\{h(X) - E[h(X)]\}^2 \\
= E\{aX + b - a\mu - b\}^2 \\
= E\{a^2[X - \mu]^2\} \\
= a^2E(X - \mu)^2 \\
= a^2V(X). \]

6. Problem 3.42.
(a) Because
\[ E[X(X - 1)] = E[X^2 - X] = E[X^2] - E(X) \]
\[ \Rightarrow E(X^2) = E[X(X - 1) + E(X)] = 27.5 + 5 = 32.5. \]
V(X) = E(X^2) - [E(X)]^2 = 32.5 - 5^2 = 7.5.

(c) It is clear that

E[X(X - 1)] = E(X^2) - E(X) = V(X) + [E(X)]^2 - E(X).

7. Problem 3.46. The probability function is

(a) For v6,

\[ b(3; 8, 0.6) = \binom{8}{3} 0.6^3 (1 - 0.6)^{8-3} = 56 \times 0.216 \times 0.0102 = 0.1234. \]

For v7,

\[ b(3; 8, 0.35) = \binom{8}{3} 0.35^3 (1 - 0.35)^{8-3} = 56 \times 0.04288 \times 0.1160 = 0.2785. \]

(b) \[ b(5; 8, 0.6) = \binom{8}{5} 0.6^5 (1 - 0.6)^{8-5} = 56 \times 0.07776 \times 0.064 = 0.2787. \]

(c) If you use version 6, the answer is

\[ P[3 \leq X \leq 5] = b(3; 8, 0.6) + b(4; 8, 0.6) + b(5; 8, 0.6) \]

\[ = 0.1234 + 0.2322 + 0.2787 \]

\[ = 0.6343. \]

If you use version 7, the answer is

\[ P[3 \leq X \leq 5] = b(3; 7, 0.6) + b(4; 7, 0.6) + b(5; 7, 0.6) \]

\[ = 0.1935 + 0.2903 + 0.2613 \]

\[ = 0.7451. \]

(d) If you use version 6, the answer is

\[ P(1 \leq X) = 1 - P(X = 0) = 1 - 0.9^{12} = 0.7175. \]

If you use version 7, the answer is

\[ P(1 \leq X) = 1 - P(X = 0) = 1 - 0.9^9 = 0.6126. \]

8. Problem 3.49.

(a) Let X be the number of second within those six. Then,

\[ P(X = 1) = \binom{6}{1} 0.1^1 0.9^5 = 0.3543. \]

(b) Let X be the number of second within those six. Then,

\[ P(X \geq 2) = 1 - P(X \leq 1) = 1 - \binom{6}{0} 0.9^6 - \binom{6}{1} 0.1^1 0.9^5 = 0.1143. \]
(c) Let $X$ be the number of times in the examination. Then,

$$P(X \leq 5) = 0.9^4 + \binom{4}{1}0.1 \times 0.9^4 = 0.9185.$$ 


The probability for a replacement is $0.2 \times 0.4 = 0.08$. Let $X$ be the number of being replaced under warranty. Then, we have

$$X \sim Bin(10, 0.08).$$

Thus, we have

$$p(2) = \binom{10}{2}0.08^20.92^8 = 0.1478.$$