

Statistical Methods (STAT 511)
Midterm #2, Spring 2009
8:00-10:00pm, LYNN 1136
Monday, March 30 2009

There are totally 28 points in the exam. The students with score higher than or equal to 25 points will receive 25 points. Please write down your name, student ID number, and mark your division below.

NAME: _____

ID: _____

DIVISION 1: ___ 8:30-9:20MWF

DIVISION 3: ___ 9:30-10:20WMF

1. (6 points). The amount of cola dispensed by a cola machine in a bottle follows a normal distribution with expected value 520(ml) and standard deviation 10(ml).

(a) (2 points). Suppose the label of the bottle states the volume of cola is 500. Compute the probability that the label of the bottle overstates its volume (i.e. the volume in the bottle is less than 500).

(b) (2 points). Suppose the volume of the bottle is 540. Compute the probability that the bottle will overflow.

(c) (2 points). Determine the volume of a new bottle so that the probability of overflow for the new bottle is less than 0.001.

2. (4 points). Flip a die 900 times. Let X be the total number of six observed. Then exactly, X follows a binomial distribution. Use normal approximation for binomial distribution.

(a) (2 points). Assume the die is balanced. Compute $P(130 \leq X \leq 170)$.

(b) (2 points). Assume the die is unbalanced with probability 0.16 equal to 6. Compute $P(X \geq 180)$.

3. (8 points). Suppose the joint PMF of X and Y is

	Y			
x	1	2	3	4
1	0.05	0.10	0.16	0.22
2	0.07	0.15	0.15	0.10

(a) (2 points). Compute $E(X)$, $V(X)$, $E(Y)$ and $V(Y)$.

(b) (2 points). Compute $Cov(X, Y)$ and $Corr(X, Y)$.

(c) (1 point). Compute $Cov(2X + 3, 5Y + 6)$ and $Corr(2X + 3, 5Y + 6)$

(d) (1 point). Is X and Y independent. Explain.

(e) (2 points). Compute the conditional PMF of Y given $X = 2$.

4. (6 points). Suppose X_1, \dots, X_n are identically independently distributed (iid) with common expected value μ and variance σ^2 . Let $\bar{X} = \sum_{i=1}^n X_i/n$. Use the Central Limit Theorem.

(a) (2 points). Suppose X_i describes the height of a person with $\mu = 176\text{cm}$ and $\sigma = 15\text{cm}$. Compute $P(172 < \bar{X} < 180)$ when $n = 100$.

(b) (2 points). Suppose X_i describes the number of patients in a hospital with $\mu = 270$ and $\sigma = 30$. Compute $P(260 \leq \bar{X} \leq 280)$ when $n = 100$.

(c) (2 points). Compute $P(2 \leq \bar{X} \leq 2.5)$ when $n = 100$ if the PMF of X_i is given by

x	1	2	3
$p(x)$	0.25	0.25	0.50

5. (4 points). Compute the following probabilities by using properties of the linear combination of normal distribution. Assume independence.

(a) (2 points). Suppose $X_1 \sim N(1, 1)$, $X_2 \sim N(1.5, 1.2)$ and $X_3 \sim N(1.5, 1.4)$.
Compute $P(X_1 + X_2 > 2X_3)$.

(b) (2 points). Suppose $X_1, X_2 \sim N(0, 1)$ and $X_3, X_4 \sim N(0.5, 1)$.
Compute $P(X_1 + X_2 - X_3 - X_4 > 2.0)$.