1. Problem 2.6.1.

Solution: The PDF of $X$ is

$$f(x) = \frac{1}{R-L}, L < x < R$$

and the CDF of $X$ is

$$F(x) = \frac{x-L}{R-L}, L < x < R.$$ 

Then, the CDF of $Y$ is

$$F_Y(y) = P(Y \leq y) = P(cX + d \leq y) = P(X \leq (y-d)/c) = \frac{(y-d)/c - L}{R-L} = \frac{y - (cL+d)}{(cR+d) - (cL+d)}.$$ 

Therefore, $Y \sim \text{Uniform}[cL+d, cR+d]$.

2. Problem 2.6.3.

Solution: The CDF of $X$ is

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt.$$ 

The CDF of $Y$ is

$$F_Y(y) = P(cX + d \leq y) = P(X \leq (y-d)/c)$$

$$= \int_{-\infty}^{(y-d)/c} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$

$$= \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\sigma c} e^{-\frac{(t-\mu-c\sigma)^2}{2\sigma^2}} ds$$

$$= \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\sigma c} e^{-\frac{(t-\mu+c\sigma)^2}{2\sigma^2}} ds,$$

where $s = (t-d)/c$. Therefore, $Y \sim \mathcal{N}(c\mu + d, c^2\sigma^2)$.

3. Problem 2.6.4.

Solution: The CDF of $X$ is

$$F(x) = e^{-\lambda x}, x > 0.$$ 

The CDF of $Y = cX$ for a positive $c$ is

$$F_Y(y) = P(cX \leq y) = P(X \leq y/c) = e^{-\lambda y/c} = e^{-(\lambda/c)y}, y > 0.$$ 

Therefore $Y \sim \text{Exponential}(\lambda/c)$.

4. Problem 2.7.9.

(a) Solution: For $0 < x < 2$, there is

$$f_X(x) = \int_{0}^{2} f_{X,Y}(x,y)dy = \frac{1}{4} \int_{x}^{2} (x^2 + y)dy = \frac{1}{4} (x^2y + \frac{y^2}{2}) \bigg|_{x}^{2} = -\frac{x^3}{4} + \frac{3x^2}{8} + \frac{1}{2}.$$ 

(b) Solution: For $0 < y < 2$, there is

$$f_Y(y) = \int_{0}^{2} f_{X,Y}(x,y)dx = \frac{1}{4} \int_{0}^{y} (x^2 + y)dx = \frac{1}{4} (\frac{x^3}{3} + xy) \bigg|_{0}^{y} = \frac{y^3}{12} + \frac{y^2}{4}.$$
5. 2.8.8.

Solution:

\[ P(Y = 5) = \int_0^\infty P(Y = 5|X = x)2e^{-2x}dx = \int_0^\infty 2e^{-3x}e^{-2x}dx = 2 \int_0^\infty e^{-5x}dx = 0.4. \]

6. 2.8.12.

Solution:

\[ P(X = 1|Y = 5) = P(X = 1) = 1/3. \]