1.

a. P(X = 1, Y = 1) = p(1,1) = .20

b. P(X ≤ 1 and Y ≤ 1) = p(0,0) + p(0,1) + p(1,0) + p(1,1) = .42

c. At least one hose is in use at both islands. P(X ≠ 0 and Y ≠ 0) = p(1,1) + p(1,2) + p(2,1) + p(2,2) = .70

d. By summing row probabilities, $p_x(x)$ = .16, .34, .50 for x = 0, 1, 2, and by summing column probabilities, $p_y(y)$ = .24, .38, .38 for y = 0, 1, 2. P(X ≤ 1) = $p_x(0) + p_x(1)$ = .50

e. P(0,0) = .10, but $p_x(0) \cdot p_y(0)$ = (.16)(.24) = .0384 ≠ .10, so X and Y are not independent.

2.

a.

		y					
	p(x,y)	0	1	2	3	4	
	0	.30	.05	.025	.025	.10	.5
x	1	.18	.03	.015	.015	.06	.3
	2	.12	.02	.01	.01	.04	.2
		.6	.1	.05	.05	.2	

b. P(X ≤ 1 and Y ≤ 1) = p(0,0) + p(0,1) + p(1,0) + p(1,1) = .56
= (.8)(.7) = P(X ≤ 1) · P(Y ≤ 1)

c. P(X + Y = 0) = P(X = 0 and Y = 0) = p(0,0) = .30

d. P(X + Y ≤ 1) = p(0,0) + p(0,1) + p(1,0) = .53

7.

a. p(1,1) = .030

b. P(X ≤ 1 and Y ≤ 1 = p(0,0) + p(0,1) + p(1,0) + p(1,1) = .120

c. P(X = 1) = p(1,0) + p(1,1) + p(1,2) = .100; P(Y = 1) = p(0,1) + … + p(5,1) = .300

d. P(overflow) = P(X + 3Y > 5) = 1 – P(X + 3Y ≤ 5) = 1 – P[(X,Y)=(0,0) or …or (5,0) or (0,1) or (1,1) or (2,1)] = 1 - .620 = .380

e. The marginal probabilities for X (row sums from the joint probability table) are $p_x(0)$ = .05, $p_x(1)$ = .10 , $p_x(2)$ = .25, $p_x(3)$ = .30, $p_x(4)$ = .20, $p_x(5)$ = .10; those for Y (column sums) are $p_y(0)$ = .5, $p_y(1)$ = .3, $p_y(2)$ = .2. It is now easily verified that for every (x,y), $p(x,y) = p_x(x) \cdot p_y(y)$, so X and Y are independent.

22.

a. E(X + Y) = $\sum_x \sum_y (x+y)p(x,y) = (0+0)(.02)$

$$+(0+5)(.06)+...+(10+15)(.01) = 14.10$$

b. E[max (X,Y)] = $\sum_x \sum_y \max(x+y)\cdot p(x,y) = (0)(.02)+(5)(.06)+\ldots+(15)(.01) = 9.60$

26. Revenue = 3X + 10Y, so E (revenue) = E (3X + 10Y)

$$= \sum_{x=0}^{5}\sum_{y=0}^{2}(3x+10y)\cdot p(x,y) = 0\cdot p(0,0)+\ldots+35\cdot p(5,2) = 15.4$$

29. Cov(X,Y) = $-\frac{2}{75}$ and $\mu_x = \mu_y = \frac{2}{5}$. E($X^2$) = $\int_0^1 x^2 \cdot f_x(x)dx$

$= 12\int_0^1 x^3(1-x^2dx) = \frac{12}{60} = \frac{1}{5}$, so Var (X) = $\frac{1}{5} - \frac{4}{25} = \frac{1}{25}$

Similarly, Var(Y) = $\frac{1}{25}$, so $\rho_{X,Y} = \frac{-2/75}{\sqrt{\frac{1}{25}}\cdot\sqrt{\frac{1}{25}}} = -\frac{50}{75} = -.667$

30.

a. E(X) = 5.55, E(Y) = 8.55, E(XY) = (0)(.02) + (0)(.06) + … + (150)(.01) = 44.25, so Cov(X,Y) = 44.25 – (5.55)(8.55) = -3.20

b. $\sigma_X^2 = 12.45, \sigma_Y^2 = 19.15$, so $\rho_{X,Y} = \frac{-3.20}{\sqrt{(12.45)(19.15)}} = -.207$