CHAPTER 4

1.

a. P(X ≤ 1) = $\int_{-\infty}^{1} f(x)dx = \int_0^1 \frac{1}{2}xdx = \frac{1}{4}x^2\Big]_0^1 = .25$

b. P(.5 ≤ X ≤ 1.5) = $\int_{.5}^{1.5} \frac{1}{2}xdx = \frac{1}{4}x^2\Big]_{.5}^{1.5} = .5$

c. P(X > 1.5) = $\int_{1.5}^{\infty} f(x)dx = \int_{1.5}^{2} \frac{1}{2}xdx = \frac{1}{4}x^2\Big]_{1.5}^{2} = \frac{7}{16} \approx .438$

8.

c. P(Y ≤ 3) = $\int_0^3 \frac{1}{25}ydy = \frac{y^2}{50}\Big]_0^3 = \frac{9}{50} \approx .18$

d. P(Y ≤ 8) = $= \int_0^5 \frac{1}{25}ydy + \int_5^8 (\frac{2}{5} - \frac{1}{25}y)dy = \frac{23}{25} \approx .92$

e. P(3 ≤ Y ≤ 8) = P(Y ≤ 8) – P(Y < 3) = $\frac{46}{50} - \frac{9}{50} = \frac{37}{50} = .74$

f. P(Y < 2 or Y > 6) = $= \int_0^2 \frac{1}{25}ydy + \int_6^{10} (\frac{2}{5} - \frac{1}{25}y)dy = \frac{2}{5} = .4$

11.

a. P(X ≤ 1) = F(1) = $\frac{1}{4} = .25$

b. P(.5 ≤ X ≤ 1) = F(1) – F(.5) = $\frac{3}{16} = .1875$

c. P(X > .5) = 1 – P(X ≤ .5) = 1 – F(.5) = $\frac{15}{16} = .9375$

f. E(X) = $\int_{-\infty}^{\infty} x \cdot f(x)dx = \int_0^2 x \cdot \frac{1}{2}xdx = \frac{1}{2}\int_0^2 x^2dx = \frac{x^3}{6}\Big]_0^2 = \frac{8}{6} \approx 1.333$

g. E(X^2) = $\int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^2 x^2 \frac{1}{2}xdx = \frac{1}{2}\int_0^2 x^3dx = \frac{x^4}{8}\Big]_0^2 = 2,$

So Var(X) = E(X^2) – [E(X)]^2 = $2 - \left(\frac{8}{6}\right)^2 = \frac{8}{36} \approx .222$, $\sigma_x \approx .471$

14.

a. If X is uniformly distributed on the interval from A to B, then

$$E(X) = \int_A^B x \cdot \frac{1}{B-A}\,dx = \frac{A+B}{2}, E(X^2) = \frac{A^2 + AB + B^2}{3}$$

V(X) = E(X^2) – [E(X)]^2 = $\frac{(B-A)^2}{2}$.

With A = 7.5 and B = 20, E(X) = 13.75, V(X) = 13.02

c. P(X ≤ 10) = F(10) = .200; P(10 ≤ X ≤ 15) = F(15) – F(10) = .4

d. σ = 3.61, so $\mu \pm \sigma$ = (10.14, 17.36)
Thus, P($\mu - \sigma \le X \le \mu + \sigma$) = F(17.36) – F(10.14) = .5776
Similarly, P($\mu - 2\sigma \le X \le \mu + 2\sigma$) = P(6.53 ≤ X ≤ 20.97) = 1